Competition, human capital and income inequality with limited commitment[1]

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Abstract

We develop a dynamic model with two-sided limited commitment to study how barriers to competition, such as restrictions to *business start-up* and *non-competitive covenants*, affect the incentive to accumulate human capital when contracts are not enforceable. High barriers lower the outside value of 'skilled workers' and reduce the incentive to accumulate human capital, particularly when contracts are not enforceable. In contrast, low barriers can result in over-accumulation of human capital. This can be socially optimal if there are positive spillovers. A calibration exercise shows that this mechanism can account for a sizable portion of cross-country income inequality.

JEL classification: J24, J41, O30

Key words: Contract enforcement, barriers to competition, human capital, economic growth.

> "*Wages rise quickly and sometimes dramatically in response to increases in the market demand for certain type of labor, but these increases are a reaction to competition from other firms, not to internal pressure from employees.*"
>
> Bewley (1999, p. 407)

1 Introduction

As Bewley reminds us, labor market conditions 'external' to the firm play a crucial role in the determination of wages. Since wages determine the return from human capital, external conditions are also crucial for the incentive of workers to accumulate human capital. In this paper we show that 'external competitive' conditions are important if contracts are not enforceable neither for the employers nor for the employees (double sided limited commitment).

It is not difficult to see why in reality contracts are not fully enforceable for both employers and employees. On the one hand, without slavery or serfdom, workers are free to move in order to obtain the best return for their human capital. Together with the limited observability of workers' effort, this reduces the enforcement of contracts for workers. On the other, employers could renege the compensation promised to the workers for the effort to accumulate human capital. Under these circumstances, workers would be discouraged from exercising effort unless their human capital can be easily redeployed outside the firm. Since this depends on the competitiveness of the labor market, competition plays a crucial role in affecting the accumulation of human capital. The analysis of this mechanism, which was already informally described by Adam Smith but up to now has not been extensively explored, is the central focus of this paper.

We use a dynamic general equilibrium model where contracts are not enforceable, neither for workers nor for firms (employers). We shows that the way limited enforcement affects the accumulation of 'general' human capital depends on barriers to the mobility of skilled labor. In particular, high barriers discourage the accumulation of human capital while low barriers have a stimulating effect, both in new and incumbent firms. As a result, differences in 'barriers to competition' translate into significant differences in incomes and welfare across economies.

While this result vindicates the common wisdom that 'barriers to the external conditions of the firm' are 'barriers to riches', it also shows that this effect can be greatly amplified when contracts are not enforceable for both parties. In fact, a key finding of the paper is that the limited enforcement of contracts matters for the accumulation of human capital only when the limited commitment is double-sided, that is, when both sides can renege the contract. Instead, with the commitment of at least one party, barriers to the mobility of skilled labor have limited effects on the accumulation of human capital and aggregate output.

The central mechanism through which barriers to the mobility of skilled labor affect the accumulation of 'general' human capital is to reduce the outside option of skilled workers, that is, the value of redeploying their skills outside the firm. With a lower outside value, workers do not have a credible mechanism for punishing the firm from reneging the promised compensation. Anticipating this, the worker provides less effort to acquire the skills, and therefore, we have a typical time-inconsistency problem. It is important to emphasize that the analysis focuses on the type of human capital that is not specific to an individual firm, and therefore, it can be transferred to other firms. Otherwise, the outside value of the worker would be independent of human capital.

Why do we need double sided-limited commitment? If the worker could commit to staying with the firm and provide effort (one-sided commitment from the worker), the contractual friction associated with the limited commitment of the firm could be resolved by paying the worker in advance. However, without the commitment from the worker, advance payments or ex-post payments conditional on the realized productivity are not incentive-compatible. This is because the worker can simply exercise a lower effort and quit. On the other hand, if the investor could commit, the promised payments would not be reneged ex-post. Thus, it becomes feasible to implement an incentive-compatible wage mechanism to extract the right level of effort and prevent the worker from quitting. For example, in line with existing Contract Theory, one could solve the commitment problem with an output-sharing agreement or by transferring total or partial ownership of assets to the workers (e.g. Hart and Moore (1994)). But with two-sided lack of commitment, such arrangements are still open to unverifiable *de facto* renegotiations or skimming.

The only way to make the contract free from renegotiation is by choosing a level of investment in human capital that makes the *ex-ante* payment promised by the firm exactly equal to the *ex-post* outside value of the worker, making renegotiation moot. In our economy, the best outside value for the worker is the one received by entering into a contractual arrangement with a new firm. Therefore, a credible investment policy for an incumbent firm is to mimic the investment decision of a new firm. However, when the investment cost of new firms is high, that is, there are high barriers, their investment is low, implying that the investment of incumbent firms is also low. In contrast, with full or one-sided commitment, incumbent firms do not have to mimic the investment decisions of new firms. In summary, while in an economy with full or one-sided commitment barriers to competition affect only the human capital accumulation of new firms, with two-sided limited commitment they affect the investment decisions of *all* firms. In particular, in economies where in equilibrium there is no exit or entry, changing the degree of competitiveness may have no effect if there is one-sided limited commitment while the effects could be substantial if there is two-sided limited commitment.

Our results are first illustrated with a simple two-stage model which is then extended to a dynamic infinite horizon set-up. The parametrization of the infinite horizon model allows us to quantify the ability of one particular barrier—*start-up costs*—to account for different levels of human capital accumulation, as well as cross-country income differences. The baseline model can account, roughly, for half of the cross-country income gaps with the US. Even though this number should be taken with caution, given the simplicity of the model, it shows that this mechanism can be quantitatively important, bringing a new perspective on the role of competition as a factor of growth.

In our benchmark model 'barriers to the external conditions of the firm' are given by the cost of starting a new firm (which offers the best outside option to the skilled worker). However there exist many other barriers that could generate similar qualitative results. For example, we show that *covenants* (preventing a skilled-worker from working for a different employer in the same industry for a period of time) or strong enforcement of Intellectual Property Rights can depress human capital accumulation and possibly explain regional and national income differences.

The results of this paper can also be interpreted as saying that *barriers* to competition determine cross-country positions relative to the 'technology possibility frontier', without emphasizing a distinction between innovation and technology adoption. This is consistent with the idea that even the implementation of known technologies requires appropriate human capital. In contrast, Acemoglu, Aghion, and Zilibotti (2006) develop a theory where the 'distance to the frontier' determines a country's comparative advantage on innovation vs. adoption. While in their theory the cost of barriers depends on the position of a country relative to the frontier, in our framework it is the barriers that determine the position of a country in relation to the frontier. The causality effect is reversed and the policy implications are different. They show that the lack of pro-competitive policies becomes more costly as countries approach the world technology frontier, while our theory implies that the lack of pro-competitive policies can determine a country's position away from the frontier.

Amaral and Quintin (2009) also show that limited enforcement of contracts can have a large impact on equilibrium output because it reduces the capital directed to the production sector and the employment of efficient technologies. The main mechanism relies on the accumulation of 'physical' capital when contracts are not enforceable for entrepreneurs (one-sided limited enforcement). Our paper, instead, focuses on the accumulation of 'human' capital when there is double-sided limited enforcement and emphasizes the central role played by 'competition' for human capital.

This paper relates to different strands of literature. In addition to the studies already cited, at least three more should be mentioned. First, the extensive literature on growth theories based on endogenous human capital accumulation (e.g. Nelson and Phelps (1996) and Lucas (2002)). In these theories, human capital is rewarded through the usual channel of higher competitive wages. We take a closer look at this channel, showing how it can be affected by the interplay between competition and commitment in the skilled labor market. Second, the labor literature that studies the accumulation of skills either within the firm (e.g. Acemoglu and Pischke (1999)) or ex-ante, before workers and firms are matched and wages are negotiated. In most of this literature (e.g. Acemoglu (1997), Acemoglu and Shimer (1999)), higher outside values worsens the hold-up problem leading to lower accumulation of skills. In our framework stronger competition results in higher outside values[1]. Third, the literature that emphasizes the role of 'barriers to riches' in explaining income differences (Mokyr (1990) and Parente and Prescott (2002)). We have emphasized the importance of barriers to labor mobility and how the effects of these barriers depend on contractual commitment features.

[1] deMeza and Lockwood (2006) provide a summary of the literature. The 'hold-up' problem is usually created by the ex-ante nature of the investment decisions and the presence of switching costs. In our model, in contrast, agents decide investment after matching and there are no switching costs. See also Felli and Roberts (2002).

2 The model

Agents' types and preferences: There are two types of agents: A continuum of 'workers' of total mass 1 and a continuum of 'investors' of total mass $m > 1$.

Each worker is characterized by a level of human or knowledge capital h_t which is endogenous in the model. The worker's lifetime utility is $\sum_{t=0}^{\infty} \beta^t (c_t - e_t)$, where c_t is consumption and e_t is the 'effort' to accumulate knowledge. The lifetime utility of investors is $\sum_{t=0}^{\infty} \beta^t c_t$.

We assume that workers do not save. Therefore, physical capital, which is needed in production, must be provided by investors. The goal of this assumption is to differentiate the roles of workers and investors: the first as providers of human capital and the second of financial resources.[2] Given the linear specification of the utility function, the equilibrium interest rate is equal to the intertemporal discount rate; that is, $r = 1/\beta - 1$.

Technology: Production requires the input of human or knowledge capital, h_t, and physical capital, k_t. They generate output according to:

$$f(k_t, h_t) = h_t^{1-\alpha} k_t^{\alpha}.$$

Investment in knowledge or human capital, $h_{t+1} - h_t$, requires effort from the worker. The effort cost function is denoted by:

$$e_t = \varphi(h_t, h_{t+1}; H_t),$$

where H_t is the economy-wide knowledge. The dependence on the aggregate knowledge captures possible leakage or spillover effects.

The function φ is strictly decreasing in H_t and h_t, strictly increasing and convex in h_{t+1}, and satisfies $\varphi(h_t, h_t; H_t) > 0$. The last condition implies that human capital depreciates if the agent does not exercise enough effort. It is further assumed to be homogeneous of degree $\rho > 1$. With this homogeneity assumption we have a semi-endogenous growth model as in Jones (1995)

[2]If workers were allowed to save, they would self-finance the purchase of physical capital eliminating the contractual frictions between investors and workers. Zero savings could also be interpreted as an endogenous outcome if workers discount more heavily than investors. As long as the discount differential is sufficiently high, workers will not save in equilibrium. See also Michelacci and Quadrini (2009).

generating long-term differences in income *levels*. The analysis can be easily extended to $\rho = 1$, in which case we would have long-term *growth* differences.[3]

Physical capital is knowledge-specific. When a worker upgrades the level of knowledge, only part of the existing capital is usable with the new knowledge. Knowledge upgrading is equivalent to the adoption of a new technology that makes part of the existing equipment economically obsolete. This is formalized by assuming that the depreciation rate of physical capital is:

$$\delta_t = \delta \cdot \left(\frac{h_{t+1}}{h_t} \right).$$

This assumption should not be interpreted as capturing an environment in which workers with higher human capital cannot use part of the existing physical capital. Rather, in order for the higher human capital to be more productive, part of the existing physical capital needs to be replaced with new capital.[4]

Because of physical capital obsolescence, there is an asymmetry between *incumbent firms*—whose capital depreciates with more advanced knowledge—and *new firms* which, without capital in place, have a greater incentive to hire workers with higher knowledge. This is a manifestation of Arrow's replacement effect which, for simplicity, we impose exogenously in the model.

Competitive structure: Investors compete to hire workers in a Walrasian market without matching frictions. They offer contracts that determine the investment in human and physical capital and the compensation structure. We refer to the contractual arrangement between an investor and a worker as a *firm*. For expositional simplicity we assume that each investor can hire only one worker. However, the investor-worker pair can also be interpreted as a specific project or unit within a large firm, all sharing certain characteristics.

[3] Our choice of a semi-endogenous model is dictated by the cross-country evidence we will discuss later. The model can be interpreted as a detrended version of an economy that grows at the 'exogenous' rate dictated by worldwide knowledge. Let $\overline{H}_t$ be worldwide knowledge growing at the exogenous rate $\bar{g}$, with the effort cost function $\tilde{\varphi}(h_t, h_{t+1}; H_t, \overline{H}_t)$ homogeneous of degree 1. After normalizing all variables by $\overline{H}_t$, the cost can be written as $\varphi(h_t, h_{t+1}; H_t)$, which is homogeneous of degree $\rho > 1$.

[4] As an example, suppose that a worker who knows how to use a standard typewriter learns how to use a ward processing software. Obviously, without the acquisition of a computer the new skills are useless. But once the computer is acquired, the old typewriter is no longer useful; it becomes economically obsolete.

First, the relationship with each worker is governed by a specific contract; second, investors behave competitively (for example, they can not collude to prevent workers' mobility); third, there is free entry and therefore, no worker remains unemployed.

In each period there is a mass 1 of investors who are in a contractual relationship with workers, and a mass $m-1>0$ who are idle and could start new firms. Investors can borrow from and lend to each other to finance physical capital k at the interest rate $r=1/\beta-1$. The labor market opens twice, before and after the accumulation of knowledge. Potential new firms funded by idle investors create a competitive demand for human capital (workers) and physical capital. Even though there is no entry in the steady state, the potential for the creation of new firms—which is guaranteed by the assumption that there are more investors than workers ($m>1$)—is crucial for the characterization of the equilibrium.

Barriers to entry: The effective competition for workers created by potential entrants is limited by several types of barriers. For the moment we consider only barriers imposed to start up firms. The analysis of other barriers, such as the strict enforcement of covenants, will be conducted in Section 5 with similar results.

Barriers to entry are modeled as a deadweight cost proportional to the initial level of knowledge chosen by the firm. Given the initial knowledge h_{t+1}, the entry cost is $\tau\cdot h_{t+1}$. This choice is motivated by its analytical convenience and the key findings of the paper are robust to alternative specifications.[5]

3 One-period model with entry barriers

Before studying the general model with infinitely lived agents, we first consider a simplified version with only one period to facilitate the intuition for the key results of the paper.

There are two stages: before and after the investment in knowledge. The states at the beginning of the period are h_0 and k_0. After making the investment decisions, h_1 and k_1, the firm generates output $y_1=h_1^{1-\alpha}k_1^{\alpha}$ in the

[5]For example, we could assume that the cost is proportional to the initial capital k_{t+1}, or to the initial output $h_{t+1}^{1-\alpha}k_{t+1}^{\alpha}$, or to the discounted flows of outputs. The basic results also apply when the entry cost is a fixed payment but of course the details of the analysis are different. The assumption of proportionality allows for a continuous impact of τ while a fixed cost would have an impact only after it has reached the prohibitive level.

second stage. In this simple version of the model we assume that physical capital fully depreciates after production. The worker receives a payment w at the end of the period, i.e. after the choice of h_1. Payments before the choice of h_1 are not incentive-compatible because of the limited enforcement of contracts for workers. With only one period, we can ignore the discounting as well as the spillovers in the cost to accumulate human capital.

The timing of the model is as follows: The firm starts with initial states h_0 and k_0. At this stage the worker decides whether to stay or quit the firm. If the worker quits, she can be hired by a new firm (funded by an idle investor). The capital owned by the incumbent firm, k_0, can be sold to the new firm at a competitive price that will be derived below. If the worker decides to stay, she will exercise effort to upgrade the knowledge capital to h_1 and the investor provides the funds to upgrade the physical capital to k_1. After the investment, the firm pays w_0. At this stage the worker can still quit and switch to a new firm. However, she cannot change the level of knowledge h_1. The investor is the residual claimant of the firm's output.

3.1 Planner allocation

As a benchmark to which we can compare the competitive allocations, we solve first for the planner's problem. Given λ the weight assigned to workers and $1-\lambda$ assigned to investors, the planner solves the problem:

$$\max_{c^w,c^i,h_1,k_1}\left\{\lambda\Big[c^w-\varphi(h_0,h_1)\Big]+(1-\lambda)c^i\right\}$$

subject to:

$$-k_1+\left[1-\delta\cdot\left(\frac{h_1}{h_0}\right)\right]k_0+h_1^{1-\alpha}k_1^{\alpha}\geq c^w+c^i$$

$$c^w-\varphi(h_0,h_1)\geq 0$$

$$c^i\geq 0,$$

where c^w and c^i are consumption for workers and investors respectively. The problem is subject to the resource constraint and the participation constraints for workers and investors. The assumption here is that consumption cannot be negative and the planner cannot force workers to put effort unless their utility is positive.

The first order conditions for knowledge and human capital are:

$$(1-\alpha)\left(\frac{h_1}{k_1}\right)^{-\alpha} = \varphi_2(h_0,h_1)+\delta\cdot\left(\frac{k_0}{h_0}\right) \tag{1}$$

$$\alpha\left(\frac{h_1}{k_1}\right)^{1-\alpha} = 1, \tag{2}$$

while the choices for consumption are determined from the resource constraint and one of the participation constraints. If $\lambda > 0.5$ ($\lambda < 0.5$), the participation constraint for investors (workers) is binding. If $\lambda = 0.5$ the planner is indifferent on the relative consumption of workers and investors.

The first order conditions have simple interpretation. The marginal products of human and physical capital (the terms on the left hand side) are equalized to the marginal costs (the terms on the right hand side). We will come back to these conditions after the characterization of the competitive allocations.

3.2 Contractual arrangements

Contracts specify the investments in human and physical capital and how the revenues will be shared, subject to the participation constraints that depend on the particular assumption about commitment. These contracts have the property that workers and investors agree on the levels of investment independently of how the net surplus is shared between the worker and the firm. To solve for the contracting problem we need to define the surpluses and outside options for workers and firms.

Given initial levels of human and physical capital, h_0 and k_0, the worker's surplus is:

$$W(h_0,k_0;h_1,w_0) = w_0-\varphi(h_0,h_1). \tag{3}$$

This is the wage (consumption) promised by the firm minus the effort cost to accumulate human capital. Although the wage is chosen in the first stage of the period, the actual payment could arise either in the first or second stage, depending on the particular commitment environment.

After the investment in knowledge, the effort cost is sunk. Furthermore, the wage received in the second stage could be different from the promised one if the firm renegotiates. Therefore, the surplus after the investment in knowledge can be written as:

$$\widehat{W}(h_1, k_1; \hat{w}_0) \;\; = \;\; \hat{w}_0, \tag{4}$$

where we use the *hat* sign to denote functions and variables that are defined *after* the investment in knowledge (second stage). We will keep this notational convention throughout the paper.

Before incurring the cost of investing in human capital, the net surplus for the worker is the payment w_0 minus the effort cost. After the investment, the effort cost becomes sunk and the surplus is only the payment $\hat{w}_0$. Also notice that the arguments of the functions include both the states and the variables that will be chosen by the contract. At the beginning of the period the states are h_0 and k_0. After the investment the states are h_1 and k_1.

The surpluses of the firm are:

$$J(h_0, k_0; h_1, k_1, w_0) \;\; = \;\; -w_0 - k_1 + \left[1 - \delta \cdot \left(\frac{h_1}{h_0}\right)\right] k_0 + h_1^{1-\alpha} k_1^{\alpha} \tag{5}$$

$$\widehat{J}(h_1, k_1; \hat{w}_0) \;\; = \;\; -\hat{w}_0 + h_1^{1-\alpha} k_1^{\alpha}. \tag{6}$$

Also for the firm we distinguish the stage before and after the investment. In the second stage the investment cost becomes sunk and the firm could pay the worker $\hat{w}_0$ instead of the promised payment w_0.

The repudiation values or outside options for the worker are the values of quitting the current employer and switching to a new firm. Because of competition among potential entrants (free entry) and absence of matching frictions, workers are always able to extract the whole surplus created by new firms. Therefore, the repudiation values are the total surpluses created by new firms. If the worker quits at the beginning of the period before the investment in knowledge this is equal to:

$$D_w(h_0) \;\; = \;\; \max_{h_1, k_1} \left\{ -\varphi(h_0, h_1) - \tau h_1 - k_1 + h_1^{1-\alpha} k_1^{\alpha} \right\}, \tag{7}$$

while the repudiation value after the investment is:

$$\widehat{D}_w(h_1) \;\; = \;\; \max_{k_1} \left\{ -\tau h_1 - k_1 + h_1^{1-\alpha} k_1^{\alpha} \right\}, \tag{8}$$

where again we have used the hat sign to denote the repudiation value after the investment in knowledge. The assumption that workers are always able to extract the whole surplus created by new firms is without loss of generality. As we show in Appendix C, similar results are obtained in the case in which the worker extracts only a share of the surplus created by new firms.

New firms create a competitive demand also for the physical capital of incumbent firms. The purchase of physical capital from an existing firm is equivalent to the acquisition of the existing firm and, through the acquisition, the new firm avoids the payment of the entry cost τh_1. The price that the new firm is willing to pay is the cost it saves from acquiring the existing firm. This includes the transferred capital and the avoidance of the entry cost. Therefore, given h_1^{New} the knowledge investment chosen by the new firm, the price paid for acquisition of an incumbent firm at the beginning of the period is:

$$D_f(k_0, h_0) \quad = \quad \left[1 - \delta \cdot \left(\frac{h_1^{New}}{h_0}\right)\right] k_0 + \tau h_1^{New}. \tag{9}$$

The value of the transferred capital is $[1 - \delta \cdot (h_1^{New}/h_0)]k_0$ since part of the capital k_0 owned by the existing firm depreciates.

After the investment in knowledge, the acquisition price is:

$$\widehat{D}_f(k_1, h_1) \quad = \quad k_1 + \tau h_1 \tag{10}$$

In the second stage the physical capital chosen by a new firm is exactly equal to the capital owned by the incumbent firm since at this point it is no longer possible to change h_1. Therefore, the transferred capital is worth k_1.

Equations (9) and (10) define the outside values for incumbent firms, before and after the accumulation of human capital. Notice that incumbent firms are able to extract the whole surplus from new firms because of the competitive structure, that is, there are many potential entrants ($m > 1$).

3.3 Equilibrium with one-sided limited commitment

We are now ready to characterize the competitive equilibrium starting with the case in which only the investor commits. It will then be trivial to show that the equilibrium investment does not change when is the worker who commits. As we will see, it is the limited commitment of both parties (double-sided limited enforcement) that induces a deviation from the optimal investment.

3.3.1 Contract when the investor commits

In the context of the one period model, commitment means that the agent does not renege the contract in the second stage after the investment in knowledge. With investor's commitment, the optimal contract can be characterized by choosing all variables at the beginning of the period through Nash bargaining. The optimal policy solves:

$$\max_{h_1,k_1,w_0} \Big[W(h_0,k_0;h_1,w_0) - D_w(h_0)\Big]^{1-\eta}\Big[J(h_0,k_0;h_1,k_1,w_0) - D_f(h_0,k_0)\Big]^{\eta}, \tag{11}$$

where η is the bargaining power of the firm.

The repudiation values, $D_w(h_0)$ and $D_f(h_0,k_0)$, are determined by the optimal policies of new firms (see (7) and (9)). Since these policies are taken as given by incumbent firms, the repudiation values are also taken as given in the bargaining problem. The functions $W(h_0,k_0;h_1,w_0)$ and $J(h_0,k_0;h_1,k_1,w_0)$ are defined in (3) and (5).[6]

Appendix A derives the first order conditions and shows that the optimal choice of human capital h_1 satisfies the condition:

$$(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} \;=\; \varphi_2(h_0,h_1) + \delta\cdot\left(\frac{k_0}{h_0}\right), \tag{12}$$

where the subscript in the cost function φ denotes the partial derivative.

The optimal policy of a new firm is determined by the first order condition to problem (7); that is,

$$(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} \;=\; \varphi_2(h_0,h_1) + \tau. \tag{13}$$

The left-hand terms in (12) and (13) are the marginal productivity of knowledge. The right-hand terms are the marginal costs. For an incumbent firm, the marginal cost derives from the effort incurred by the worker plus the obsolescence of physical capital. For a new firm the obsolescence cost is replaced by the entry cost. These two conditions clearly show the different incentives to invest for an incumbent versus a new firm. On the one hand,

[6]As long as $0 < \eta < 1$, we do not need to impose the participation constraints, that is, $W(h_0,k_0;h_1,w_0) - D_w(h_0) \geq 0$ and $J(h_0,k_0;h_1,k_1,w_0) - D_f(h_0,k_0) \geq 0$. In the limiting cases with $\eta = 1$ or $\eta = 0$, however, the problem would not be well defined without the constraints. The first order conditions, though, remain valid.

new firms do not have any physical capital and knowledge upgrading does not generate capital obsolescence. On the other, they must pay the entry cost τh_1.

It is important to point out that the relative bargaining power η does not matter for the choice of the investment in knowledge. This only matters for the division of the surplus. For incumbent firms the surplus is split according to the bargaining power η. In the case of new firms the whole surplus goes to the worker as a consequence of free-entry of new firms. However, we show in Appendix C that assigning a positive bargaining power to new firms would not change the properties of the model.

Let h_1^{Old} be the optimal knowledge investment of an incumbent old firm (the solution to condition (12)) and h_1^{New} the optimal investment of a new firm (the solution to condition (13)). The following proposition formalizes the relation between barriers to entry and knowledge investment.

Proposition 1 *With investor's commitment, the knowledge investment of an incumbent firm, h_1^{Old}, does not depend on τ. Instead, the knowledge investment of a new firm, h_1^{New}, is strictly decreasing in the entry cost τ until $h_1^{New} = \underline{h} < h_0$ and there exists $\bar{\tau} > 0$ such that $h_1^{New} = h_1^{Old}$.*

Proof 1 *As already shown in Appendix A, the first order condition of incumbent firms in the choice of k_1 is $\alpha(k_1/h_1)^{\alpha-1} = 1$. This is also the first order condition of new firms derived by differentiating in (7). Using this condition, (12) and (13) become:*

$$
\begin{aligned}
(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} &= \varphi_2(h_0, h_1^{Old}) + \delta \cdot \left(\frac{k_0}{h_0}\right) \\
(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} &= \varphi_2(h_0, h_1^{New}) + \tau.
\end{aligned}
$$

The proposition follows directly from these two conditions until h_1^{New} reaches the lower bound $\underline{h}$. This is the level of human capital when the agent does not exert effort, defined by $\varphi(h_0, \underline{h}) = 0$. Q.E.D.

In equilibrium there is no entrance of new firms and the investment in knowledge is $h_1 = h_1^{Old}$. Therefore, when the investor commits to the contract, the entry barriers τ are irrelevant for the equilibrium investment. They only affect the worker's payment. Furthermore, the investment in knowledge and physical capital are exactly the same as those chosen by a planner. In fact, the first order condition for incumbent firms, equation (12), is exactly the same as the first order condition for the planner, equation (1). The same first order conditions are also obtained for the investment in physical capital.

Appendix A also derives the worker's payment which is equal to:

$$\begin{aligned} w_0 &= (1-\eta)\cdot\left\{\left[1-\delta\cdot\left(\frac{h_1}{h_0}\right)\right]k_0 + h_1^{1-\alpha}k_1^{\alpha} - k_1 - D_f(k_0,h_0)\right\} \\ &+ \eta\cdot\Big\{\varphi(h_0,h_1)+D_w(h_0)\Big\}. \end{aligned} \tag{14}$$

The wage w_0 depends on τ since the repudiation values $D_w(h_0)$ and $D_f(k_0,h_0)$ contribute to the determination of the worker's payment. Notice that this expression also holds in the limiting cases of $\eta = 1$ and $\eta = 0$. In the first case the payment to the workers is the repudiation value plus the cost of accumulating knowledge. This means that the participation constraint for the worker is binding. When $\eta = 0$, it is the participation constraint of the investor that binds.[7]

Before continuing it is important to notice that the *ex-ante* participation constraint for the worker is satisfied whenever the *ex-post* participation constraint is satisfied. This means that the contract is enforceable if the worker has not incentive to renege after the accumulation of human capital. To see this, consider the worst case scenario in which all the bargaining power is held by firms, that is, $\eta = 1$. In this case, according to (14), the wage received by the worker in the second stage is $w_0 = \varphi(h_0,h_1) + D_w(h_0)$. If the worker quits she will receive $\widehat{D}_w(h_1)$. All we have to show is that $\varphi(h_0,h_1) + D_w(h_0) \geq \widehat{D}_w(h_1)$. In fact, from (7) and (8) we have that:

$$\begin{aligned} D_w(h_0) &= \max_{h,k}\Big\{-\varphi(h_0,h)-\tau h - k + h^{1-\alpha}k^{\alpha}\Big\} \geq \\ & \max_{k}\Big\{-\varphi(h_0,h_1)-\tau h_1 - k + h_1^{1-\alpha}k^{\alpha}\Big\} = \\ & -\varphi(h_0,h_1)+\widehat{D}_w(h_1) \end{aligned}$$

Therefore, we have that $\varphi(h_0,h_1) + D_w(h_0) \geq \widehat{D}_w(h_1)$ for an any value of h_1, that is, the payment received by a staying worker is never smaller than

[7]In our framework human capital is chosen within the contract. Therefore, in strict sense, workers do not choose h_{t+1} strategically. For a model where workers choose h_{t+1} strategically see Marimon and Rendahl (2010).

the payment she will receive by quitting in the second stage of the period. Of course, if the worker has no incentive to renege when the firm has all the bargaining power, this must also be true when the bargaining power of the worker is positive.

3.3.2 Contract when the worker commits

Does the result that investment does not depend on τ also hold when the worker commits but the investor does not commit? In this case the investor can renege on the promised payments after the worker has invested in knowledge, that is, in the second stage of the period. With worker's commitment, however, the investor's incentive to renege can be resolved by making the payment w_0 in the first stage of the contract, that is, before the investment in knowledge. As long as the contract is enforceable for the worker, there is no risk that she runs away or does not exercise the effort to acquire knowledge after receiving w_0.

Of course, if the result holds when one of the parties commits, it also holds when both parties commit to the contract. Because h_1^{Old} maximizes the total surplus, this must also be the equilibrium investment if both parties commit.

3.4 Equilibrium with double-sided limited commitment

We want to show first that, when the investor can not commit to fulfill his promises, he will renege the contract in the second stage after the choice of h_1. To see this, we must derive the value that the worker would get by quitting the firm in the second stage with knowledge h_1. This is the surplus generated by a new firm that hires the worker as defined in (10).

Once the parties renege on an existing contract, they bargain the terms of a new contract. The bargaining solution solves the problem:

$$\max_{\hat{w}_1} \Big[\widehat{W}(h_1,k_1;\hat{w}_0) - \widehat{D}_w(h_1)\Big]^{1-\eta} \Big[\widehat{J}(h_1,k_1;\hat{w}_0) - \widehat{D}_f(h_1,k_1)\Big]^{\eta} \quad (15)$$

where the surpluses in the second stage are defined in (4) and (6) and the repudiation values are defined in (8) and (10).

Taking first order conditions with respect to $\hat{w}_0$ and solving (see Appendix B), the wage received by the worker in the second stage of the period is:

$$\hat{w}_0 \;=\; (1-\eta)\Big[f(h_1,k_1) - \widehat{D}_f(k_1,h_1)\Big] + \eta \widehat{D}_w(h_1) \qquad (16)$$

We can now use the definitions of the repudiation values provided in (8) and (10) to eliminate $\widehat{D}_w(h_1)$ and $\widehat{D}_f(k_1, h_1)$ and express the wage received by the worker after renegotiation as:

$$\hat{w}_0 = f(h_1^{Old}, k_1^{Old}) - k_1^{Old} - \tau h_1^{Old} \tag{17}$$

Therefore, if in the first stage of the period the worker decides to stay with the current employer, ex-post she will receive the utility:

$$\hat{w}_0 - \varphi(h_0, h_1^{Old}) = f(h_1^{Old}, k_1^{Old}) - k_1^{Old} - \tau h_1^{Old} - \varphi(h_0, h_1^{Old}) \tag{18}$$

If instead the worker chooses to quit for a new firm in the first stage of the period, she will get the surplus created by a new firm, that is:

$$\max_{h_1,k_1} \Big\{ f(h_1, k_1) - k_1 - \tau h_1 - \varphi(h_0, h_1) \Big\} = f(h_1^{New}, k_1^{New}) - k_1^{New} - \tau h_1^{New} - \varphi(h_0, h_1^{New}) \tag{19}$$

Comparing the value received from staying—equation (18)—with the value received from quitting—equation (19)—we can see that, as long as $h_1^{Old} \neq h_1^{New}$ (and $k_1^{Old} \neq k_1^{New}$), the option of quitting at the beginning of the period dominates the option of staying with the incumbent firm and agreeing to the policy with commitment. The only way to retain the worker is for the incumbent firm to agree to the same knowledge investment chosen by a new firm; that is, h_1^{New}. In this way the worker keeps the repudiation value available at the beginning of the period and prevents the firm from reneging.[8] This is stated formally in the next proposition.

Proposition 2 *Suppose that all firms have the same initial states* (k_0, h_0)*. Then there is a unique equilibrium with aggregate knowledge* $H_1 = h_1^{New}$*, which is strictly decreasing in the entry cost* τ *until* $h_1^{New} = \underline{h} < h_0$.

Proof 2 *It is enough to show that with any* $h_1 \neq h_1^{New}$*, the worker will get a lower utility. This must be the case because the value received by a staying worker, equation (18), is maximized at* h_1^{New}*. The last part of the proposition follows from Proposition 1.*

[8]The result that incumbents firms 'mimic' the investment policy of the start-up firms is a direct consequence of the time-inconsistency problem when neither the worker nor the firm are able to commit. Sub-game perfection requires that the ex-ante promises are ex-post delivered. The only instrument that can be used to insure that the investor's promisees are delivered ex-post is by agreeing to the investment policy of start-up firms because this will determine the outside values ex-post.

Because h_1^{New} is decreasing in τ (as established in Proposition 1), the accumulation of knowledge decreases with the cost of entry. Therefore, with double-sided limited commitment, there is a negative correlation between barriers to entry and the accumulation of knowledge. This is in contrast to the equilibrium achieved when only one of the parties or both commit to the contract. In this case the accumulation of knowledge is $H_1 = h_1^{Old}$. This is also the investment chosen by the planner. As we have seen in the previous subsection, h_1^{Old} is independent of τ and, therefore, the equilibrium investment does not depend on entry barriers.[9]

To summarize, when contracts are not enforceable for both workers and investors, greater competition (lower barriers to entry) leads to higher investment in knowledge. Because the investment is determined by the optimality condition of new firms, the equilibrium investment is not necessarily efficient for incumbent firms. In particular, if τ is small, incumbent firms accumulate too much knowledge. This is because the accumulation of human capital is associated with faster economic obsolescence of physical capital in incumbent firms (Arrow's replacement effect) but not in new firms.[10]

4 The infinite horizon model

In this section we generalize the model to an infinite horizon set-up. There are two important gains from extending the analysis to the infinite horizon. First, it allows us to derive the initial conditions k_0 and h_0 endogenously as steady state values. Second, an infinite horizon structure is better suited for the quantitative analysis of Section 6.

In analyzing the infinite horizon setup, we focus on steady state equilibria. Therefore, we ignore the aggregate states as an explicit argument of the value functions. Although there is no entrance of firms in equilibrium, we still need to solve for the dynamics of a potential new firm in order to determine the outside or repudiation values for workers and firms since new firms would experience a transition to the long-term level of physical and knowledge capital.

[9] If we allow for entry and exit at the steady state equilibrium, competition will have a marginal effect on the aggregate level of human capital even in the case of one-sided limited commitment because it affects the investment decision of new entrants.

[10] As we will see in the general model, higher investment may still be socially desirable if there are positive spillovers in the accumulation of human capital.

Before proceeding, it will be convenient to define the gross output function inclusive of undepreciated capital as follows:

$$\pi(h_t, k_t, h_{t+1}) = h_t^{1-\alpha} k_t^{\alpha} + \left[1 - \delta \cdot \left(\frac{h_{t+1}}{h_t}\right)\right] k_t. \quad (20)$$

4.1 Planner's allocation

Also for the infinite horizon model we start characterizing the planner's allocation with relative weight λ. Given initial states k_t and h_t, the planner solves the problem:

$$\max_{\{c_j^w, c_j^i, k_{j+1}, h_{j+1}\}_{j=t}^{\infty}} \left\{ \lambda \sum_{j=t}^{\infty} \beta^{j-t} \Big[c_j^w - \varphi(h_j, h_{j+1}; h_j) \Big] + (1-\lambda) \sum_{j=t}^{\infty} \beta^{j-t} c_j^i \right\}$$

subject to:

$$\pi(h_j, k_j, h_{j+1}) - k_{j+1} = c_j^w + c_j^i$$

$$c_j^w - \varphi(h_j, h_{j+1}; h_j) \geq 0$$

$$c_j^i \geq 0,$$

where the constraints must be satisfied for any $j \geq t$. The variables c_j^w and c_j^i denote the consumption of workers and investors.

The first order conditions for knowledge and physical capital are:

$$-\pi_3(h_t, k_t, h_{t+1}) + \varphi_2(h_t, h_{t+1}; h_t) = \beta \Big[\pi_1(h_{t+1}, k_{t+1}, h_{t+2}) - \varphi_1(h_{t+1}, h_{t+2}; h_{t+1}) - \varphi_3(h_{t+1}, h_{t+2}; h_{t+1}) \Big] \quad (21)$$

$$\beta \pi_2(h_{t+1}, k_{t+1}, h_{t+2}) = 1 \quad (22)$$

We will come back to these conditions when we compare the planner's allocation to the competitive allocations.

4.2 Some preliminaries

Let $\mathbf{h}^t = (h_{t+1}, ...)$, $\mathbf{k}^t = (k_{t+1}, ...)$, $\mathbf{w}^t = (w_t, w_{t+1}, ...)$ be the sequences of human capital, physical capital and wages that solve the contractual problem at t. These sequences need to be derived. For the moment, however, we assume to know them. Given these sequences, we define the following surpluses for workers and firms:

$$W(h_t, k_t) = \sum_{j=t}^{\infty} \beta^{j-t} \Big[w_j - \varphi(h_j, h_{j+1}; H) \Big] \tag{23}$$

$$\widehat{W}(h_{t+1}, k_{t+1}) = w_t + \sum_{j=t+1}^{\infty} \beta^{j-t} \Big[w_j - \varphi(h_j, h_{j+1}; H) \Big] \tag{24}$$

$$J(h_t, k_t) = \sum_{j=t}^{\infty} \beta^{j-t} \Big[\pi(h_j, k_j, h_{j+1}) - w_j - k_{j+1} \Big] \tag{25}$$

$$\widehat{J}(h_{t+1}, k_{t+1}) = -w_t + \sum_{j=t+1}^{\infty} \beta^{j-t} \Big[\pi(h_j, k_j, h_{j+1}) - w_j \Big] \tag{26}$$

The hat sign denotes functions that are defined in the second stage of the period t and the arguments of the functions are the relevant state variables. For example, $W(h_t, k_t)$ is the value of the worker in the first stage of period t when the state variables are h_t and k_t. The function $\widehat{W}(h_{t+1}, k_{t+1})$ is the value of the worker in the second stage of period t, after the accumulation of human and physical capital. At this stage the states are h_{t+1} and k_{t+1}. Notice that the new investments chosen at time t, h_{t+1} and k_{t+1}, will generate output at time $t + 1$.

Keeping the assumption of competition among new entrants, the repudiation values of workers are the surpluses generated by new firms. At the beginning of period t, the surplus of a new firm, and therefore, the repudiation value of workers, is:

$$\begin{aligned} D_w(h_t) = \max_{\{k_{j+1}, h_{j+1}\}_{j=t}^{\infty}} \Bigg\{ & -\tau h_{t+1} - k_{t+1} - \varphi(h_t, h_{t+1}; H) + \\ & \sum_{j=t+1}^{\infty} \beta^{j-t} \Big[\pi(h_j, k_j, h_{j+1}) - k_{j+1} - \varphi(h_j, h_{j+1}; H) \Big] \Bigg\} \end{aligned} \tag{27}$$

After the investment in knowledge, the repudiation value is:

$$\begin{aligned} \widehat{D}_w(h_{t+1}) = \max_{k_{t+1}, \{k_{j+1}, h_{j+1}\}_{j=t+1}^{\infty}} \Bigg\{ & -\tau h_{t+1} - k_{t+1} + \\ & \sum_{j=t+1}^{\infty} \beta^{j-t} \Big[\pi(h_j, k_j, h_{j+1}) - k_{j+1} - \varphi(h_j, h_{j+1}; H) \Big] \Bigg\}. \end{aligned} \tag{28}$$

Again, this is the surplus generated by a new firm that hires a worker in the second stage of the period with knowledge capital h_{t+1}.

The key difference between a firm entering at the beginning of the period and after the investment in knowledge is that the effort to accumulate knowledge has already been exercised and h_{t+1} is given at this stage. This explains why the choice of knowledge starts in the next period.

The repudiation values of firms are given by:

$$D_f(h_t, k_t) \;=\; \pi(h_t, k_t, h_{t+1}^{New}) + \tau h_{t+1}^{New} \tag{29}$$

$$\widehat{D}_f(h_{t+1}, k_{t+1}) \;=\; k_{t+1} + \tau h_{t+1} \tag{30}$$

where h_{t+1}^{New} is the initial investment in knowledge chosen by a new firm created in the first stage of period t. This is the solution to Problem (27).

Using these functions we can now look at the special cases of limited commitment of one or both parties.

4.3 Equilibrium with one-sided commitment

We will concentrate on the case with investor's commitment. The equilibrium allocation when the worker commits (with or without commitment from the investor) is equivalent as for the one-period model.

The optimal solution for a new firm is characterized by the first order conditions to problem (27). Because of the entry cost and the obsolescence of physical capital, the optimality conditions in the entry period differ from the optimality conditions in subsequent periods, when the firm becomes an incumbent. The first order conditions with respect to k_{t+1} and h_{t+1} when the firm is started are:

$$\tau + \varphi_2(h_t, h_{t+1}; H) = \beta\Big[\pi_1(h_{t+1}, k_{t+1}, h_{t+2}) - \varphi_1(h_{t+1}, h_{t+2}; H)\Big] \tag{31}$$

$$\beta\pi_2(h_{t+1}, k_{t+1}, h_{t+2}) = 1, . \tag{32}$$

The first condition equalizes the gross marginal return of capital to its marginal cost, which is 1. The last condition equalizes the marginal cost of accumulating knowledge to the discounted value of its return (greater production and lower cost of future knowledge investment).

The first order conditions after entering are:

$$-\pi_3(h_t, k_t, h_{t+1}) + \varphi_2(h_t, h_{t+1}; H) = \beta\Big[\pi_1(h_{t+1}, k_{t+1}, h_{t+2}) - \varphi_1(h_{t+1}, h_{t+2}; H)\Big] \tag{33}$$

$$\beta\pi_2(h_{t+1}, k_{t+1}, h_{t+2}) = 1, \tag{34}$$

together with a transversality condition. This is the optimality condition for an incumbent firm, that is, a firm that was created in previous periods.

Conditions (31) and (33) show the asymmetry between new and incumbent firms. While the marginal benefit from investing in knowledge (the right-hand side) is the same, the marginal cost (the left-hand side) differs. For new firms this includes the entry cost, τ. For incumbent firms the entry cost is replaced by the depreciation of physical capital, $-\pi_3(h_t, k_t, h_{t+1})$.

We can now characterize the steady state. Let's provide first a formal definition of a steady state equilibrium.

Definition 1 *A steady state equilibrium with investor's commitment is given by constant values of knowledge H and physical capital K that solve the first order conditions (33)-(34), that is,*

$$-\pi_3(H, K, H) + \varphi_2(H, H; H) = \beta\Big[\pi_1(H, K, H) - \varphi_1(H, H; H)\Big] \tag{35}$$

$$\beta\pi_2(H, K, H) = 1. \tag{36}$$

Because in equilibrium there is no entrance, all firms have the economy-wide knowledge H. The convergence to the economy-wide average is guaranteed by the spillovers in the accumulation of knowledge: firms with lower than average knowledge tend to invest more. Thanks to the complementarity between knowledge and capital, all firms accumulate the same economy-wide level of physical capital K. In particular, as we prove in Appendix D:

Proposition 3 *There is a unique steady state equilibrium in which all firms have the same knowledge H and physical capital K.*

After solving for H and K, we can solve for the steady state payment w. This requires finding the whole transition experienced by a 'new firm', as characterized by the first order conditions (31)-(34). Even if in equilibrium workers do not quit and new firms are not created, the payment w must satisfy the enforcement constraints which depend on the surplus created by new firms, that is, $W(H, K; \mathbf{s}) \geq D_w(H)$ and $\widehat{W}(H, K; w, \mathbf{s}) \geq \widehat{D}_w(H, K)$.

Conditions (35) and (36) also reveal that the entry cost τ does not affect the steady state values of K and H. We will see in the next section that this property does not hold when there is limited commitment from both parties. Before turning to the case with double-sided limited commitment, however, let's compare the steady state competitive allocation with the steady state of the planner's allocation. If there are not externalities, that is, $\varphi_3 = 0$, the competitive allocation (with commitment) will be equal to the planner's allocation. In fact, the first order conditions (21)-(22) are equivalent to (33)-(34). With externalities, however, the two allocations differ because of the term φ_3 that enters the first order condition of the planner is nonzero. This means that the planner takes into account the positive spillovers in the accumulation of human capital, leading to higher levels of knowledge and physical capital.

4.4 Equilibrium with double-sided limited commitment

With double-sided limited commitment, the contract is renegotiated in every period, before and after the investment in knowledge. In this subsection we show how the results of the one-period model generalize to the infinite-horizon model. In particular we show that: (*i*) workers receive lower utility if the contract is renegotiated after investing in human capital compared to the utility they would receive if they quit for a new firm at the beginning of the period before making the investment (Proposition 4); (*ii*) in order to retain their workers, incumbent firms mimic the investment policy (human and physical capital) of new firms (Corollary 1); (*iii*) the characterization of a steady state equilibrium with double-sided limited commitment requires us to solve for a fixed-point problem. This fix point problem takes into account that, after the entry period, new firms become incumbent firms, and therefore, at that point they will mimic the investment in knowledge of future new firms (Proposition 5).

When the parties renegotiate the contract, they bargain over the terms of a new contract. To study the bargaining problem, it will be convenient to rewrite the net surpluses (23)-(26) recursively:

$$W(h_t, k_t) = w_t - \varphi(h_t, h_{t+1}; H) + \beta W(h_{t+1}, k_{t+1}) \tag{37}$$

$$\widehat{W}(h_{t+1}, k_{t+1}) = \hat{w}_t + \beta W(h_{t+1}, k_{t+1}) \tag{38}$$

$$J(h_t, k_t) = \pi(h_t, k_t, h_{t+1}) - w_t - k_{t+1} + \beta J(h_{t+1}, k_{t+1}) \tag{39}$$

$$\widehat{J}(h_{t+1}, k_{t+1}) = -\hat{w}_t + \beta J(h_{t+1}, k_{t+1}) \tag{40}$$

which are defined for a given solution to the contractual problem.

Using these functions, the repudiation values for the worker (27) and (28) can be rewritten as:

$$D_w(h_t) = \max_{h_{t+1}, k_{t+1}} \Big\{ -\tau h_{t+1} - k_{t+1} - \varphi(h_t, h_{t+1}; H_t) + \beta W(h_{t+1}, k_{t+1}) + \beta J(h_{t+1}, k_{t+1}) \Big\} \tag{41}$$

$$\widehat{D}_w(h_{t+1}) = \max_{k_{t+1}} \Big\{ -\tau h_{t+1} - k_{t+1} + \beta W(h_{t+1}, k_{t+1}) + \beta J(h_{t+1}, k_{t+1}) \Big\}. \tag{42}$$

As before, these values are determined by maximizing the surplus generated by a new firm that enters in the first stage of the period (Problem 41) or in the second stage (Problem 42). For later reference we denote by $h_{t+1}^{New} = f(h_t)$ the human capital chosen by a new firm that hires a worker with knowledge capital h_t at the beginning of period t. This is the solution to Problem (41). Since a new firm does not have any physical capital, the only individual state variable is h_t.

We now have all the elements to write the bargaining problem solved by an incumbent firm in the event of renegotiation. If the contract is renegotiated at the beginning of period t, the bargaining problem solves:

$$\max_{w_t, h_{t+1}, k_{t+1}} \Big[W(h_t, k_t) - D_w(h_t) \Big]^{1-\eta} \Big[J(h_t, k_t) - D_f(h_t, k_t) \Big]^{\eta}, \tag{43}$$

with solutions denoted by w_t^{Old}, h_{t+1}^{Old} and k_{t+1}^{Old}.

If the contract is renegotiated after the investment, the actual payment to the worker will be determined by the bargaining problem

$$\max_{\hat{w}_t} \left[\widehat{W}(h_{t+1}, k_{t+1}) - \widehat{D}_w(h_{t+1})\right]^{1-\eta} \left[\widehat{J}(h_{t+1}, k_{t+1}) - \widehat{D}_f(h_{t+1}, k_{t+1})\right]^{\eta}, \quad (44)$$

which takes as given the new levels of human and physical capital.

What we want to show is that, if the contract is renegotiated after the investment in knowledge, the worker will receive less utility than the utility received by quitting the firm at the beginning of the period. This is stated in the following proposition.

Proposition 4 *If the contract is renegotiated after the investment in knowledge, a worker that stays with the firm and agrees to the investment h_{t+1}^{Old} will receive less utility than a worker that quits at the beginning of the period.*

Proof 3 *See Appendix E.*

A direct implication of this proposition is stated in the following corollary.

Corollary 1 *With double-sided limited commitment, the knowledge investment chosen by an incumbent firm is equal to the knowledge investment chosen by a new firm; that is, $h_{t+1}^{Old} = h_{t+1}^{New} = f(h_t)$.*

Proof 1 *It follows from the proof of Proposition 4.*

Since the firm can renege the promised payments after the investment in knowledge, the worker would not stay unless the firm agrees to the same knowledge investment chosen by a new firm. In this way, the worker keeps the outside value high and prevents the firm from reneging.

Let's now characterize the optimal contract taking into account the above corollary. The optimization problem solved by an incumbent firm at the beginning of period t is given by

$$\max_{w_t, k_{t+1}} \left\{ \left[w_t - \varphi(h_t, f(h_t); H) + \beta W(f(h_t), k_{t+1}) - D_w(h_t)\right]^{1-\eta} \right. \quad (45)$$
$$\left. \times \left[\pi(h_t, k_t, f(h_t)) - w_t - k_{t+1} + \beta J(f(h_t), k_{t+1}) - D_f(h_t, k_t)\right]^{\eta} \right\},$$

which is obtained by substituting (37) and (39) in the bargaining problem (43) and imposing $h_{t+1} = f(h_t)$. Notice that in this problem only w_t and k_{t+1} are chosen since h_{t+1} will be determined by the function $f(h_t)$. This guarantees that the contract will not be renegotiated after the investment in knowledge, as established in Corollary 1.

It is easy to see that in this problem the wage does not affect the choice of capital. Therefore, we will focus on the optimal choice k_{t+1} which satisfies the first order condition:

$$\beta \pi_2(h_{t+1}, k_{t+1}, h_{t+2}) = 1. \tag{46}$$

This is equivalent to the first order condition when there is commitment (see equation (34)).

We denote by $g(h_t, k_t) = k_{t+1}$ the physical capital chosen by an incumbent firm with states h_t and k_t, which satisfies the first order condition (46).

Incumbent firms take as given the investment policy of new firms, $f(h_t)$. Therefore, in order to characterize the optimal policy of incumbent firms, we need to characterize the policy of new firms, to which we now turn.

A new firm that enters in period t becomes an incumbent firm starting in the next period. Therefore, starting from period $t+1$, the firm solve exactly the same problem solved by any incumbent firm. Therefore, a firm that enters in the first stage of period t solves the following problem:

$$\max_{h_{t+1}, k_{t+1}} \Bigg\{ -\tau h_{t+1} - k_{t+1} - \varphi(h_t, h_{t+1}; H) + \sum_{j=t+1}^{\infty} \beta^{t-j} \Big[\pi(h_j, k_j, h_{j+1}) - k_{j+1} - \varphi(h_j, h_{j+1}; H) \Big] \Bigg\} \tag{47}$$

subject to

$$h_{j+1} = f(h_j), \qquad \text{for } j > t.$$
$$k_{j+1} = g(h_j, k_j), \qquad \text{for } j > t.$$

Only the initial knowledge h_{t+1} and physical capital k_{t+1} are chosen in this problem. Future values are determined by the policies $h_{j+1} = f(h_j)$ and $k_{j+1} = g(h_j, k_j)$, that is, the policies of incumbent firms.[11]

[11] We can also reduce the arguments of the function determining physical capital as

The solution for h_{t+1} characterizes the optimal knowledge investment chosen by new firms, that is, the function $f(h_t)$. Since this choice depends on the function $f(.)$ determining the knowledge investment of new firms in future periods, we have a fixed point problem. This can also be seen from the first order conditions. As shown in Appendix F, the first order conditions are given by:

$$\tau + \varphi_2(h_t, h_{t+1}; H) = \beta\Big\{\pi_1\Big(h_{t+1}, k_{t+1}, f(h_{t+1})\Big) - \varphi_1\Big(h_{t+1}, f(h_{t+1}); H\Big) + \Big[\pi_3\Big(h_{t+1}, k_{t+1}, f(h_{t+1})\Big) + \tau\Big] f_1(h_{t+1})\Big\} \quad (48)$$

$$\beta\pi_2(h_{t+1}, k_{t+1}, f(h_{t+1})) = 1. \quad (49)$$

While the optimality condition for the choice of capital is the same as in the case of commitment (see (32)), the condition for the choice of knowledge is different (see (31)). In particular we have the unknown function $f(h_t)$ that enters the first order condition. Then, to determine this function, we need to solve a fixed point problem.

Denote by $h_{t+1} = \psi(h_t; f)$ the policy function that solves problem (47), for a given policy $f(h_t)$. A steady state equilibrium is defined as follows:

Definition 2 *A steady state equilibrium with double-side limited commitment is given by a function $f(h_t)$, a mapping $\psi(h_t; f)$, and constant values of knowledge H and physical capital K such that:*

(i) The mapping $\psi(h_t; f)$ solves problem (47) given $f(h_t)$ and satisfies the first order condition (48);

(ii) The function $f(h_t)$ is the fixed point of the mapping $\psi(h_t; f)$, that is, $f(h_t) = \psi(h_t; f)$;

(iii) H and K satisfy conditions (48) and (49), that is,

$$\tau + \varphi_2(H, H; H) \ = \ \beta\Big\{\pi_1\Big(H, K, f(H)\Big) - \varphi_1\Big(H, f(H); H\Big) \ +$$

follows: $k_{j+1} = g(h_j, k_j) = \tilde{g}(h_j)$. We can do this by using the first order condition (46), that is, $\beta\pi_2(f(h_t), k_{t+1}, f(f(h_t))) = 1$, which implicitly defines $k_{t+1} = \tilde{g}(h_t)$.

$$f_1(H)\left[\pi_3\Big(H,K,f(H)\Big)+\tau\right]\Bigg\} \quad (50)$$

$$\beta\pi_2(H,K,H)=1. \quad (51)$$

In the above definition of a steady state equilibrium we do not specify the distribution of the surplus between workers and firms since such distribution, which depends on the relative bargaining power, does not affect investment decisions. In the presentation of the contractual problems we assumed that workers have the whole bargaining power when hired by new firms but we presented the renegotiation problem in incumbent firms for any relative bargaining power. This asymmetry is without loss of generality. As we show in Appendix C, the assumption that workers have the same bargaining power in new and incumbent firms will result in the same investments in human and physical capital.

Because incumbent firms innovate at the same rate as new firms, condition (48) also determines the investment in knowledge of incumbent firms. Therefore, in order to determine whether the lack of commitment leads to higher or lower investment in knowledge, we have to compare condition (50) to the steady state condition when the investor commits to the long-term contract, that is, condition (35).

Let H^C be the steady state knowledge in the economy in which the investor commits, and H^{NC} the steady state knowledge without commitment. We have the following proposition:

Proposition 5 *Suppose that $f_1(H) \leq 1$. Then the steady state value of H^{NC} is strictly decreasing in τ and there exists $\bar{\tau} > 0$ such that $H^{NC} > H^C$ for $\tau < \bar{\tau}$ and $H^{NC} < H^C$ for $\tau > \bar{\tau}$.*

Proof 4 *See Appendix G.*

Notice that the proof is based on the assumption that $f_1(H) \leq 1$; that is, the derivative of the policy function at the steady state equilibrium is not greater than one. Although we could not establish this property analytically, we have checked the condition numerically in all quantitative exercises conducted in the paper and found to be satisfied.[12]

To summarize, when contracts are not enforceable neither for the worker nor for the investor, barriers to entry are harmful for the accumulation of knowledge. With low barriers, the economy experiences a higher level of income than in the case with commitment. This could be welfare-improving if there are spillovers in the accumulation of knowledge.

5 Covenants and other barriers to mobility

Other barriers to mobility such as *covenants* (preventing a skilled-worker from working for a different employer in the same industry for a period of time), can be incorporated in our model to account for regional differences. For example, the evolution of the computer industry exemplifies the effects of both types of *barriers* to competition. As Bresnahan and Malerba (2002) emphasize, this industry has gone through different technological stages (from main-frames to PCs and the Internet). Knowledge in this particular industry was geographically spread in many countries including Europe. Yet the United States has persistently been the industry leader. According to them, this dominance can be explained by "*...the existence of a large body of technical expertise in universities and the generally supportive environment for new firm formation in the United States*", Bresnahan and Malerba (2002, page 69).

While lower barriers to business start-up may have favored the computer leadership of the United States, different enforcement of covenants—and informational linkages across firms—may have determined the shift of regional leadership within the United States. As argued by Saxenian (1996), Gilson (1999) and Hyde (2003), Silicon Valley dominates over Route 128 due to a Californian legal and social tradition of not enforcing post-employment covenants, resulting in high labor mobility and knowledge spillovers.

A natural way to model non-competitive covenants is by assuming that a quitting worker can use only a fraction ξ of the accumulated knowledge in a new firm. This formulation also captures the case in which part of the knowledge can not be used by the worker due to the enforcement of IPR if

[12]The numerically solution is based on the iteration over the functions $f(h)$ and $f_1(h)$. We start by guessing these two functions after approximating them with piece-wide linear functions over a grid for h. Given the guesses for $f(h)$ and $f_1(h)$, we find the values of h' that satisfy condition (50) at each grid point. The solutions are then used to update the guesses for $f(h)$ and $f_1(h)$ until convergence.

she does not have full control of the patent. As Boldrin and Levine (2008) have forcefully argued, such form of IPR enforcement may deter innovation. In our formulation, a more stringent enforcement of covenants (or IPRs) is captured by a lower ξ.

To keep the presentation brief, we limit the analysis to the one-period model. The extension to an infinite horizon will follow the same logic as in the analysis with entry costs. The problem solved by a new firm which started at the beginning of the period can be written as:

$$D_w(h_0) = \max_{h_1,k_1}\left\{ -\varphi(h_0,h_1) - k_1 + (\xi h_1)^{1-\alpha}k_1^{\alpha}\right\} \tag{52}$$

The problem solved by an incumbent firm is as in problem (11). The first order conditions with respect to h_1, for incumbent and new firms respectively, are:

$$(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} = \varphi_2(h_0,h_1) + \delta\cdot\left(\frac{k_0}{h_0}\right) \tag{53}$$

$$(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} = \varphi_2(h_0,h_1)\cdot\xi^{\alpha-1}. \tag{54}$$

Because $\xi < 1$ and $\alpha < 1$, the term $\xi^{\alpha-1} > 1$. Therefore, covenants have the effect of increasing the cost of accumulating knowledge and act similarly to the entry cost τ. Proposition 1 becomes:

Proposition 6 *The knowledge investment of a new firm h^{New} is strictly increasing in ξ and there exists $\bar{\xi} > 0$ such that $h^{New} = h^{Old}$.*

Proof 5 *Using the first order condition for the choice of physical capital, which is $\alpha(k_1/h_1)^{\alpha-1} = 1$ for both incumbent and new firms, the above first order conditions can be rewritten as:*

$$(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} = \varphi_2(h_0,h^{Old}) + \delta\cdot\left(\frac{k_0}{h_0}\right)$$

$$(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} = \varphi_2(h_0,h^{New})\xi^{-1}$$

The proposition follows directly from these two conditions. *Q.E.D.*

All the results obtained in Section 3 trivially extend to the case of covenants and other similar barriers to the mobility of knowledge.

6 Cross-country evidence on barriers to business start-up and quantitative application

In this section we first show that there is a cross-country negative relation between the cost of business start-up—which in our theory is one of the barriers to knowledge mobility—and income. We then calibrate the model to assess the contribution of this particular barrier to cross-country income inequality.

A publication from the World Bank (2005) provides indicators of the quality of the business environment for a cross-section of countries, including proxies for barriers to business start-up. The indicators are based on official sources (from government publications, reports of development agencies such as the World Bank and USAID, and government web pages on the Internet). Absent official documents, the study takes the estimates of local incorporation lawyers.

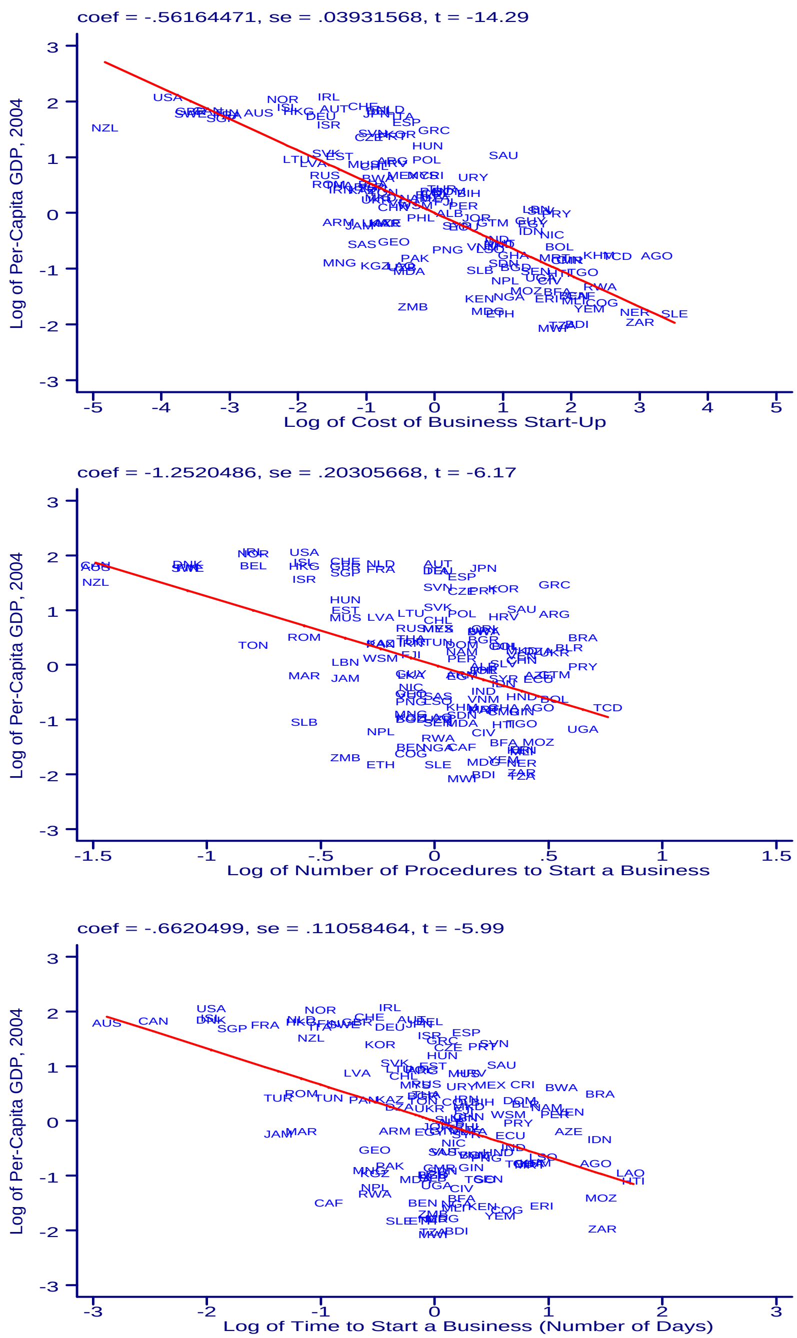
Figure 1 plots the level of per-capita GDP in 2004 against three of these proxies: the 'cost of starting a new business', the 'number of bureaucratic procedures' that need to be filed before starting a new business, and the average 'length of time' required to start a new business.[13] All variables are in log.

As can be seen from the figure, there is a strong negative association between the economic development of a country and barriers to business start-up. Although the negative correlation does not imply causation, it suggests that our theory could potentially explain some of the income differences across countries.

To investigate this possibility we conduct a calibration exercise. After choosing the parameters, we feed into the model the actual data on the cost of business start-up for each of the country we have in the sample. The equilibrium output predicted by the model allows us to quantify the importance of the cost of business start-up for cross country income inequality. It is important to emphasize that we use the 'cost of business start-up' because of

[13] The cost of starting a new business is the average pecuniary cost needed to set up a corporation in the country, in percentage of the country per-capita income. The normalization of the cost of business start-up by the level of per-capita income better captures the importance of barriers to business start-up than the absolute dollar cost. What is relevant is the comparison between the cost of business start-up and the value of creating a business. Although the dollar cost is on average higher in advanced economies, the value of a new business is also likely to be higher.

Figure 1: Barriers to business start-up and level of development.



data availability. However, our theory applies more broadly to other barriers affecting the mobility of knowledge as those discussed in the previous section.

Apart from the cost of business start-up, which we take from the data for each country in the sample, we calibrate the economy to the United States. The discount factor, β, the production parameter, α, and the depreciation parameter, δ, are calibrated to replicate the following moments: an interest rate of 5 percent, a capital income share of 33 percent, and a capital-output ratio of 3. This implies $\beta = 0.9524$, $\alpha = 0.33$, and $\delta = 0.06$. Notice that the three moments are invariant to the entry barrier τ, and therefore, they are constant across countries.[14]

The effort cost function is derived from the accumulation equation for the stock of knowledge, which is assumed to take the form:

$$h_{t+1} = (1-\phi)h_t + \left(H_t^{\theta} e_t^{1-\theta}\right)^{\nu},$$

where H_t is the average level of knowledge, e_t is the effort cost of accumulating knowledge and ϕ is the depreciation rate. The parameter $\nu < 1$ captures the return to scale in the accumulation of knowledge and $\theta < 1$ the leakage or spillover effects. Inverting, we get the cost function:

$$e_t \;=\; \varphi(h_t, h_{t+1}; H_t) \;=\; \frac{\left[h_{t+1} - (1-\phi)h_t\right]^{\frac{1}{(1-\theta)\nu}}}{H_t^{\frac{\theta}{1-\theta}}},$$

which is homogeneous of degree $\rho = (1-\theta\nu)/(1-\theta)\nu$.

The depreciation of knowledge results from working directly with the stationary version of the model, detrended by the rate of worldwide knowledge. The parameter ϕ is then approximately equal to the exogenous rate of growth.[15] Assuming that the economy grows at 1.8 percent per year, we

[14]While it is easy to see the mapping between the first two moments and the first two parameters ($\beta = 1/(1+r)$ and $\alpha = rK/Y$), less obvious is the mapping between δ and the capital-income ratio. From condition(32), evaluated at the steady state, we have $\beta\pi_2(H,K,H) = \beta[1-\delta+\alpha(K/H)^{\alpha-1}] = 1$. Given the output function $Y = H^{1-\alpha}K^{\alpha}$, the capital-output ratio can be written as $K/Y = (K/H)^{1-\alpha}$. Using this expression to eliminate K/H in the previous condition, we get $\beta[1-\delta+\alpha/(K/Y)] = 1$. Therefore, after choosing β and α, the parameter δ is uniquely determined by the capital-output ratio.

[15]The original (undetrended) function for the accumulation of knowledge is $h_{t+1} = h_t + \overline{H}_t^{1-\nu}(H_t^{\theta}e_t^{1-\theta})^{\nu}$, where $\overline{H}_t$ is the worldwide knowledge, external to an individual country, which grows at the constant rate $\bar{g}$. Normalizing all terms by $\overline{H}_t$, the investment function becomes $h_{t+1} = (1-\phi)h_t + A(H_t^{\theta}e_t^{1-\theta})^{\nu}$, where $\phi = \bar{g}/(1+\bar{g}) \simeq \bar{g}$ and $A = 1/(1+\bar{g})$. Because A acts as a rescaling factor, we can set $A = 1$.

set $\phi = 0.018$. This is about the average growth rate in per-capita GDP experienced by the US during the last century.

The values of the other two parameters, θ and ν, are more controversial. Manuelli and Seshadri (2005) uses a similar specification of the investment function, within an overlapping generation model, but without externalities. In order to generate some key properties of the life-time profile of earnings, they choose a return to scale of 0.93. This is also the value estimated by Heckman, Lochner, and Taber (1998). We use this value to calibrate ν on the assumption that there is sufficient intergenerational transmission of human capital.[16] For the baseline parametrization we also follow Manuelli and Seshadri (2005) and assume no externalities, that is, $\theta = 0$. The sensitivity analysis will clarify how the results depend on the choice of θ and ν.

6.1 Results

Figure 2 plots the values of per-capita GDP and start-up costs for a sample of 137 countries. This is the sample for which we have data on the cost of business start up. The figure also plots the steady state values of output predicted by the model. As can be seen, the cost of business start-up captures some of the variability of income across countries.

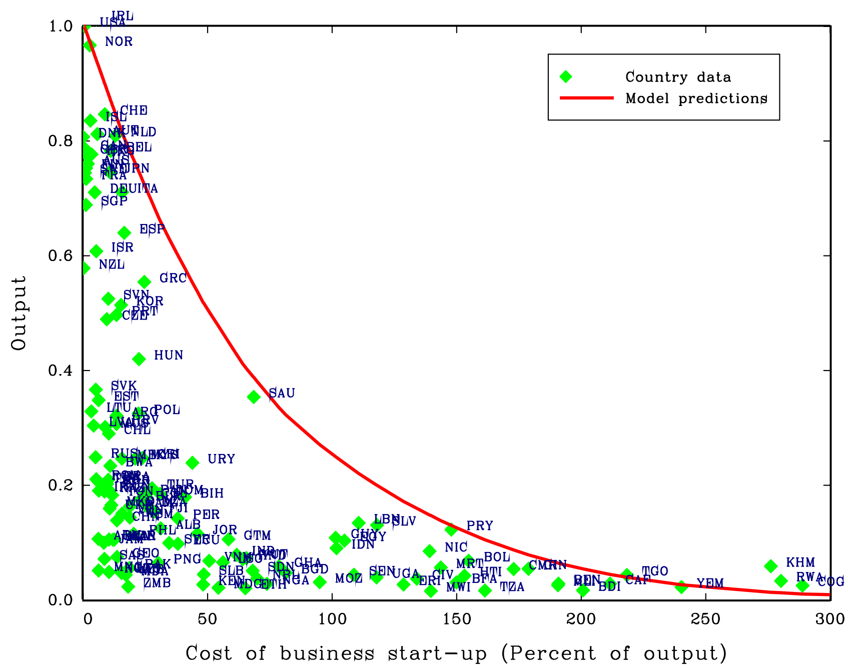
To compute the average income gap from the US captured by the model, we compute the following index:

$$\text{Index} = 1 - \frac{\sum_i |\hat{y}_i - y_i|}{\sum_i |y_{US} - y_i|},$$

where y_i is the actual income of country i, $\hat{y}_i$ is the income predicted by the model, given the observed cost of business start-up, and y_{US} is the US income. The model has been normalized so that it replicates US income; that is, $\hat{y}_{US} = y_{US}$. The index is 1 if the model replicates perfectly the actual cross-country incomes, that is, $\hat{y}_i = y_i$. It is zero if the cost of business start-up has no impact on the equilibrium income; that is, $\hat{y}_i = y_{US}$. For

[16] In Manuelli and Seshadri (2005) the cost of human capital investment has two components: time and expenditures. Our specification does not distinguish between these components and the investment cost is captured by the single variable e. However, this does not alter in important ways the main properties of the model. As Manuelli and Seshadri show, the key parameter to replicate the life-time earning profile is not the relative importance of the two inputs but the return-to-scale parameter. Notice also that the depreciation rate $\phi = 0.018$ is also equal to the value chosen by Manuelli and Seshadri.

Figure 2: Entry cost and cross-country output per capita - Data and model.



the baseline calibration the index is 0.51. Therefore, the model accounts for roughly half of the cross-country income gaps from the US.

Next we show how the values of θ and ν affect the results. Table 1 reports the income gaps accounted by the model for alternative values of these parameters. The gaps are computed using the same sample used to construct Figure 2 (which includes 137 countries).

The model accounts for larger income gaps the higher the return to scale, ν, and the lower the externalities, θ. The sensitivity is especially high for the return to scale. However, even for small returns to scale, the model accounts for a non-negligible fraction of cross-country income gaps. Even if we take the extreme parametrization chosen by Parente and Prescott (2002), $\nu = 0.6$, the model still accounts for about 11 percent of the income gaps.

We have also calculated the 'domestic socially optimal' steady-state level of output, that is, the output resulting from solving the problem of a country's planner who is not constrained by enforcement issues. The output resulting from the policies chosen by the planner differs from the competitive output

Table 1: Income gaps accounted by the model.

	Value of ν					
	0.97	*0.93*	*0.90*	*0.80*	*0.70*	*0.60*
Value of θ						
0.0	0.68	**0.51**	0.42	0.26	0.18	0.13
0.1	0.66	0.48	0.40	0.25	0.17	0.12
0.2	0.64	0.46	0.37	0.23	0.15	0.11
0.3	0.62	0.43	0.35	0.21	0.14	0.10
0.4	0.59	0.40	0.32	0.19	0.12	0.09

because of the externality at the domestic level, represented by θ. It also differs from the 'global socially optimal' steady-state level of output, which is the solution to the problem of a 'global planner', which internalizes the worldwide leakage, or spillover, represented by ν.

The steady-state values of H and K in the domestic planner allocation are found by solving the first order conditions (21) and (22) after imposing $h_t = h_{t+1} = h_{t+2} = H$ and $k_t = k_{t+1} = k_{t+2} = K$. It can be noticed that the planner's conditions are similar to conditions (36) and (35) except for the additional term $\varphi_3(H, H; H)$. This term captures the externality taken into account by the planner but ignored by the atomistic agents. In absence of externalities, that is, $\varphi_3(H, H; H) = 0$, the planner allocation in the steady state will have exactly the same knowledge and capital as in the steady state of the competitive allocation with commitment (of at least one party).

Table 2 reports the 'competitive' output as a fraction of the 'domestic socially optimal' output when there are no barriers to entry. A value greater than 1 means that there is over-accumulation of knowledge compared to the socially optimal level. As expected, this arises when the spillovers are small or zero; that is, when θ is small. In this case moderate barriers to business start-up would be welfare improving. On the other hand, values smaller than 1 mean that there is under-accumulation of knowledge compared to the socially optimal level. In this case barriers to entry are always suboptimal, while moderate subsidies could improve welfare.

Table 2: Steady-state output when contracts are not enforceable and there are no barriers to entry. Numbers are relative to the socially optimal output.

	Value of ν					
	0.97	*0.93*	*0.90*	*0.80*	*0.70*	*0.60*
Value of θ						
0.0	1.81	1.28	1.18	1.08	1.04	1.03
0.1	0.80	0.92	0.95	0.99	1.00	1.00
0.2	0.41	0.71	0.80	0.92	0.96	0.98
0.3	0.25	0.58	0.70	0.88	0.94	0.97
0.4	0.17	0.51	0.64	0.84	0.92	0.96

7 Conclusion

We have developed a theory in which *barriers* to the mobility of skilled workers affect the accumulation of human capital or knowledge, and therefore, the level of income. The theory does not simply say that competition enhances income. First, it emphasizes that some forms of limited enforcement are intrinsic to competitive labor markets, where wages are determined by demand and supply conditions. Second, it shows how different forms of contract enforcement affect the relation between competition, accumulation of human capital and economic development. In particular, when both investors and workers can not commit to long-term contracts, the accumulation of human capital is determined by those firms that value human capital the most; in our benchmark model are start-up firms.

In this way our theory captures the Bewley's view that "*Wages rise quickly and sometimes dramatically in response to increases in the market demand for certain type of labor, but these increases are a reaction to competition from other firms, not to internal pressure from employees.*", Bewley (1999, p. 407). Our contribution is to show that 'competition from other firms' is also important for the accumulation of human capital. In particular, high levels of human capital are associated with low barriers to the mobility of knowledge because lower barriers increase the 'competition from other firms'.

Using a semi-endogenous growth model, we have shown that *barriers to business start-up* have the potential to explain significant cross-country income differences. It may seem that accounting for 50% of the cross-country income gaps from the US overestimates the real contribution of our mechanism since the model is silent about many other features that are important for developing economies. Nevertheless, the fact that investment in human capital—even at the school or pre-school level—are determined by expectations of future income returns, which in turn are affected by competitive and contractual conditions, seems a powerful mechanism.

We have also shown that other *barriers to knowledge mobility*, such as strict enforcement of Covenants or Intellectual Property Rights, can have similar effects. Although we have modeled 'on-the-job human capital accumulation' and abstracted from skill-specific jobs, it should be clear that the mechanism described here is also relevant for 'before-the-job human capital accumulation'. In particular, the returns to education, or to acquiring a specific mix of general and specific skills, depend on the expected life-time returns. Our model shows that these returns depend crucially on the interplay between competition for skills in the labor market and the degree of commitment and enforcement.

A First order conditions for problem (11)

The first order condition with respect to h_1, k_1 and w_0 are:

$$-(1-\eta)\Gamma_J\,\varphi_2(h_0,h_1)+\eta\Gamma_W\left[-\delta\left(\frac{k_0}{h_0}\right)+(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha}\right]=0 \quad (55)$$

$$\eta\left(\frac{\Gamma_W}{\Gamma_J}\right)^{1-\eta}\left[-1+\alpha\left(\frac{k_1}{h_1}\right)^{\alpha-1}\right]=0 \quad (56)$$

$$(1-\eta)\Gamma_J-\eta\Gamma_W=0 \quad (57)$$

where

$$\Gamma_W = \Big[W(h_0,k_0;h_1,w_0)-D_w(h_0)\Big]$$

$$\Gamma_J = \Big[J(h_0,k_0;h_1,k_1,w_0)-D_f(h_0,k_0)\Big]$$

and the functions $W(h_0,k_0;h_1,w_0)$, $J(h_0,k_0;h_1,k_1,w_0)$, $D_w(h_0)$ and $D_f(h_0,k_0)$ are defined in (3), (5), (7) and (9).

Using condition (57) to eliminate Γ_W and Γ_J in condition (55) and re-arranging we get:

$$(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} = \varphi_2(h_0,h_1)+\delta\cdot\left(\frac{k_0}{h_0}\right)$$

which is the condition reported in (12).

Solving condition (57) for the wage we get:

$$\begin{aligned} w_0 &= (1-\eta)\cdot\left\{\left[1-\delta\cdot\left(\frac{h_1}{h_0}\right)\right]k_0+h_1^{1-\alpha}k_1^{\alpha}-k_1-D_f(k_0,h_0)\right\} \\ &+ \eta\cdot\Big\{\varphi(h_0,h_1)+D_w(h_0)\Big\} \end{aligned}$$

which is the expression reported in (14).

B First order conditions for problem (15)

The first order condition with respect to $\hat{w}_1$ is:

$$(1-\eta)\Big[\widehat{J}(h_1,k_1;\hat{w}_0)-\widehat{D}_f(h_1,k_1)\Big]=\eta\Big[\widehat{W}(h_1,k_1;\hat{w}_0)-\widehat{D}_w(h_1)\Big] \quad (58)$$

where the functions $\widehat{W}(h_1, k_1; \hat{w}_0)$, $\widehat{J}(h_1, k_1; \hat{w}_0)$, $\widehat{D}_w(h_1)$ and $\widehat{D}_f(h_1, k_1)$ are defined in (4), (6), (8) and (10).

Solving for the wage we get:

$$\hat{w}_0 = (1-\eta)\Big[f(h_1, k_1) - \widehat{D}_f(k_1, h_1)\Big] + \eta \widehat{D}_w(h_1)$$

which is the expression reported in (16).

C Bargaining over the surplus of new firms

We generalize the results of the one period model to the case in which the surplus generated by new firms is also bargained. With respect to Section 3, the only change is the determination of the outside value of workers.

If a new firm can extract part of the surplus, a free entry equilibrium can exist only if new firms have to incur a cost before entering the bargaining stage. This would be the case, for example, if there is a cost of posting a vacancy. Here we assume that there is an ex-ante cost, and therefore, in equilibrium firms must be able to extract part of the ex-post surplus to cover the cost. Notice that this cost is different from τh_1 which is paid only if the firm reaches an agrement.

Another necessary assumption is that the worker can switch employer (rematch) only once in each subperiod. Otherwise, the threat of switching again would allow the worker to get the whole surplus and the new firm will not be able to recoup the sunk cost. Given these assumptions, an equilibrium in which workers get only a share of the surplus is possible.

We will concentrate on the case of double-sided limited commitment. The case with one-sided limited commitment or full commitment is not relevant because, as we have seen, the outside value does not affect the investment in human capital chosen by incumbent firms.

The bargaining problem solved by a new firm created after the investment in knowledge is:

$$\max_{w_0, k_1} \left\{ w_0^{1-\zeta} \Big[-w_0 - k_1 - \tau h_1 + f(h_1, k_1) \Big]^{\zeta} \right\}, \tag{59}$$

where ζ is the bargaining power of a new firm. We distinguish it from the bargaining power of incumbent firms, η, to make the analysis more general. Notice that the value of not reaching an agrement for the worker is zero because at this stage she can not re-match with another employer for the second time.

The repudiation value can also be written as:

$$\widehat{D}_w(h_1) \quad = \quad (1-\zeta) \max_{k_1} \left\{ -\tau h_1 - k_1 + h_1^{1-\alpha} k_1^{\alpha} \right\}, \tag{60}$$

that is, the worker's share of the total surplus created by a new firm. The repudiation value for firms is as in (10).

Following the same logic used in the baseline model, the wage renegotiated by an incumbent firm is:

$$\hat{w}_0 = (1-\eta)\Big[f(h_1^{Old}, k_1^{Old}) - \widehat{D}_f(k_1^{Old}, h_1^{Old})\Big] + \eta \widehat{D}_w(h_1^{Old}). \tag{61}$$

Using the definition of $\widehat{D}_w(h_1)$ and $\widehat{D}_f(k_1, h_1)$ and noticing that in (60) the solution for physical capital is k_1^{Old} when $h_1 = h_1^{Old}$, the wage received by the worker can be written as:

$$\hat{w}_0 = (1-\zeta\eta)\Big[f(h_1^{Old}, k_1^{Old}) - k_1^{Old} - \tau h_1^{Old}\Big]. \tag{62}$$

Therefore, if the worker decides to stay with the current employer at the beginning of the period, she will receive the utility:

$$\hat{w}_0 - \varphi(h_0, h_1^{Old}) = (1-\zeta\eta)\Big[f(h_1^{Old}, k_1^{Old}) - k_1^{Old} - \tau h_1^{Old}\Big] - \varphi(h_0, h_1^{Old}).$$

This should be compared with the utility that the worker will receive if she quits for a new firm at the beginning of the period. Also a new firm can renege the contract in the second stage. Therefore, the worker that matches with a new firm at the beginning of the period and agrees to the policy h_1 and k_1, anticipates that the utility she will receive (given the renegotiation) is:

$$\hat{w} - \varphi(h_0, h_1) = (1-\zeta\eta)\Big[f(h_1, k_1) - k_1 - \tau h_1\Big] - \varphi(h_0, h_1).$$

Obviously, the worker will agree to stay only if the investment maximizes this utility. If the firm does not agree to this investment, the worker will choose the optimal h_1 and re-match with a new firm in the second stage. Hence, the utility obtained by quitting at the beginning of the period is:

$$\max_{h_1, k_1}\Big\{(1-\zeta\eta)\Big[f(h_1, k_1) - k_1 - \tau h_1\Big] - \varphi(h_0, h_1)\Big\} = \\ (1-\zeta\eta)\Big[f(h_1^{New}, k_1^{New}) - k_1^{New} - \tau h_1^{New}\Big] - \varphi(h_0, h_1^{New}),$$

where h_1^{New} is the knowledge investment that maximizes the utility of the worker.

As long as $h_1^{Old} \neq h_1^{New}$ (and $k_1^{Old} \neq k_1^{New}$), the value of quitting at the beginning of the period is higher than the value of staying with the incumbent firm and agreeing to the policy with commitment h_1^{Old}. The only way to retain the worker is for the incumbent firm to agree to the same knowledge investment chosen by a new firm; that is, h_1^{New}. Therefore, Proposition 2 still applies.

D Proof of Proposition 3

Consider condition (35), which we rewrite here as follows:

$$\varphi_2(H,H;H) + \beta\varphi_1(H,H;H) = \pi_3(H,K,H) + \beta\pi_1(H,K,H).$$

The right-hand term remains constant for any value of H. In fact, taking into account the functional form of π (see equation (20)), we have $\pi_3(H,K,H) = -\delta(K/H)$ and $\pi_1(H,K,H) = \delta(K/H) + (1-\alpha)(K/H)^\alpha$. These two terms only depend on the ratio K/H. From condition (36) we have $\beta\pi_2(H,K,H) = \beta[1 + \alpha(K/H)^{\alpha-1}] = 1$, which uniquely determines the ratio K/H.

Let us now look at the left-hand term. Because φ is homogenous of degree $\rho > 1$, the derivatives φ_1 and φ_2 are homogeneous of degree $\rho - 1$. Therefore, the left-hand-side term can be written as

$$\varphi_2(H,H;H) + \beta\varphi_1(H,H;H) = \Big[\varphi_2(1,1;1) + \beta\varphi_1(1,1;1)\Big] H^{\rho-1}.$$

Because $\rho > 1$, this term is strictly increasing in H, converges to zero as $H \to 0$ and to infinity as $H \to \infty$. Therefore, there exists a unique value of H that solves this condition. The uniqueness of H then implies the uniqueness of K. *Q.E.D.*

E Proof of Proposition 4

Using (38) and (40), Problem (44) can be rewritten as:

$$\max_{\hat{w}_t} \Big[\hat{w}_t + \beta W(h_{t+1}, k_{t+1}) - \widehat{D}_w(h_{t+1})\Big]^{1-\eta} \Big[-\hat{w}_t + \beta J(h_{t+1}, k_{t+1}) - \widehat{D}_f(h_{t+1}, k_{t+1})\Big]^{\eta},$$

Taking the first order condition and solving for the wage we get:

$$\hat{w}_t = (1-\eta)\Big[\beta J(h_{t+1}, k_{t+1}) - \widehat{D}_f(h_{t+1})\Big] - \eta\Big[\beta W(h_{t+1}, k_{t+1}) - \widehat{D}_w(h_{t+1})\Big].$$

After substituting the repudiations values defined in (30) and (42), the wage can be written as:

$$\hat{w}_t = \beta J(h_{t+1}, k_{t+1}) - k_{t+1} - \tau h_{t+1}.$$

Hence, the ex-post utility of the worker given the investment h_{t+1}^{Old} and k_{t+1}^{Old} is:

$$-\varphi(h_t, h_{t+1}^{Old}) + \hat{w}_t + \beta W(h_{t+1}^{Old}, k_{t+1}^{Old}).$$

Substituting the wage derived above, this can be written as:

$$-\varphi(h_t, h_{t+1}^{Old}) - k_{t+1}^{Old} - \tau h_{t+1}^{Old} + \beta J(h_{t+1}^{Old}, k_{t+1}^{Old}) + \beta W(h_{t+1}^{Old}, k_{t+1}^{Old}). \qquad (63)$$

This utility should be compared to the utility received if the worker quits the current employer before the investment in knowledge. In this case the worker would get:

$$\max_{h_{t+1}} \Big\{ -\varphi(h_t, h_{t+1}) + \widehat{D}(h_{t+1}) \Big\}.$$

Substituting the definition of the repudiation value provided in (42), this can be written as:

$$\max_{h_{t+1},k_{t+1}} \Big\{ -\varphi(h_t, h_{t+1}) - k_{t+1} - \tau h_{t+1} + \beta J(h_{t+1}, k_{t+1}) + \beta W(h_{t+1}, k_{t+1}) \Big\}, \quad (64)$$

with solutions h_{t+1}^{New} and k_{t+1}^{New}.

The comparison of (63) and (64) makes clear that the value of quitting (equation (64)) is bigger than the value of staying (equation (63)) unless $h_{t+1}^{Old} = h_{t+1}^{New}$ and $h_{t+1}^{Old} = k_{t+1}^{New}$. *Q.E.D.*

F First order conditions for problem (47)

Since Problem (47) becomes recursive starting at $t+1$, it can be rewritten as:

$$\max_{h_{t+1},k_{t+1}} \Bigg\{ -\tau h_{t+1} - k_{t+1} - \varphi(h_t, h_{t+1}; H) + \beta S(h_{t+1}, k_{t+1}) \Bigg\} \quad (65)$$

where the function $S(.,.)$ is defined recursively as:

$$S(h,k) = \pi\Big(h, k, f(h)\Big) - g(h,k) - \varphi\Big(h, f(h); H\Big) + \beta S\Big(f(h), g(h,k)\Big) \quad (66)$$

The recursive formulation takes into account that the investment in knowledge h' is determined by the function $f(h)$ and the investment in physical capital k' is determined by the function $g(h,k)$.

The first order conditions in Problem (65) are:

$$\tau - \varphi_2(h_t, h_{t+1}) + \beta S_1(h_{t+1}, k_{t+1}) = 0 \quad (67)$$

$$-1 + \beta S_2(h_{t+1}, k_{t+1}) = 0 \quad (68)$$

The derivatives S_1 and S_2 are obtained from the envelope conditions of Problem (66). Differentiating with respect to h and k, we obtain:

$$\begin{aligned} S_1(h,k) &= \pi_1(h,k,f(h)) + \pi_3(h,k,f(h))f_1(h) - g_1(h,k) - \varphi_1(h,f(h)) - \\ &\quad \varphi_2(h,f(h))f_1(h) + \beta S_1(f(h),g(h,k))f_1(h) + \beta S_2(f(h),g(h,k))g_1(h,k) \end{aligned} \quad (69)$$

$$S_2(h,k) = \pi_2(h,k,f(h)) - g_2(h,k) + \beta S_2(f(h),g(h,k))g_2(h,k) \quad (70)$$

Let's observe that the function $g(h,k) = k'$ must satisfy the optimality condition (46) for the choice of capital made by incumbent firms, that is,

$$\beta\pi_2(f(h), g(h,k), f(f(h))) = 1. \tag{71}$$

If we differentiate this condition with respect to k we get:

$$\beta\pi_{22}(f(h), g(h,k), f(f(h)))g_2(h,k) = 0.$$

Because $\pi_{22}(f(h), g(h,k), f(f(h))) \neq 0$, we must have that $g_2(h,k) = 0$. Using this result, the envelope condition (70) becomes

$$S_2(h,k) = \pi_2(h,k,f(h)). \tag{72}$$

Now consider the term $-g_1(h,k) + \beta S_2(f(h), g(h,k))g_1(h,k)$ in the envelope condition (69). Using (72) updated by one period, this terms becomes $-g_1(h,k)[1-\beta\pi_2(f(h), g(h,k), f(f(h)))]$, which must be zero given condition (71). Therefore, the envelope condition (69) can be rewritten as:

$$\begin{aligned} S_1(h,k) \quad = \quad & \pi_1(h,k,f(h)) + \pi_3(h,k,f(h))f_1(h) - \varphi_1(h,f(h)) \\ & -\varphi_2(h,f(h))f_1(h) + \beta S_1(f(h), g(h,k))f_1(h) \end{aligned} \tag{73}$$

Now let's consider the function $f(h) = h'$. Because this is the solution to the optimal investment in knowledge of new firms, it must satisfy condition (67), that is, $\tau - \varphi_2(h, f(h)) + \beta S_1(f(h), k') = 0$. We can then use this condition to eliminate $S_1(f(h), k')$ in the envelope condition (69) which can be rewritten as:

$$S_1(h,k) \quad = \quad \pi_1\Big(h,k,f(h)\Big) - \varphi_1\Big(h, f(h); H\Big) + \Big[\pi_3\Big(h,k,f(h)\Big) + \tau\Big]\, f_1(h).$$

Using this expression at $t+1$ to eliminate $S_1(h_{t+1}, k_{t+1})$ in (67), we can express the first order condition for the optimal choice of knowledge of new firms as:

$$\begin{aligned} \tau + \varphi_2(h_t, h_{t+1}; H) \quad = \quad & \beta\Bigg\{\pi_1\Big(h_{t+1}, k_{t+1}, f(h_{t+1})\Big) - \varphi_1\Big(h_{t+1}, f(h_{t+1}); H\Big) + \\ & \Big[\pi_3\Big(h_{t+1}, k_{t+1}, f(h_{t+1})\Big) + \tau\Big]\, f_1(h_{t+1})\Bigg\}, \end{aligned}$$

which is the expression reported in (48).

The optimality condition for the choice of physical capital is obtained by updating (70) and substituting in (68). This gives:

$$\beta\pi_2(h_{t+1}, k_{t+1}, h_{t+2}) = 1,$$

which is equivalent to (49).

G Proof of Proposition 5

In the steady state without commitment, potential new firms start with the same knowledge H as incumbents. Because $H = f(H)$, (50) can be written as:

$$\tau + \varphi_2(H, H; H) = \beta\Big[\pi_1(H, K, H) - \varphi_1(H, H; H)\Big] + \beta f_1(H)\Big[\pi_3(H, K, H) + \tau\Big],$$

which determines the steady state knowledge for incumbent and new firms when the investor does not commit (double-sided limited enforcement).

This condition must be compared to the optimality condition that determines the steady state knowledge when the investor commits to the contract (one-side limited enforcement). This is given by equation (35), which we rewrite as:

$$\varphi_2(H, H; H) = \beta\Big[\pi_1(H, K, H) - \varphi_1(H, H; H)\Big] + \pi_3(H, K, H).$$

The homogeneity of degree ρ of the cost function φ implies that the derivatives are homogeneous of degree $\rho - 1$. Therefore, the above two conditions can be rewritten as:

$$\Big[\varphi_2(1, 1; 1) + \beta\varphi_1(1, 1; 1)\Big]H^{\rho-1} = \beta\pi_1(H, K, H) + \beta f_1(H)\pi_3(H, K, H) - \tau\Big[1 - \beta f_1(H)\Big] \quad (74)$$

$$\Big[\varphi_2(1, 1; 1) + \beta\varphi_1(1, 1; 1)\Big]H^{\rho-1} = \beta\pi_1(H, K, H) + \pi_3(H, K, H). \quad (75)$$

Because $\rho - 1 > 0$, the left-hand terms are strictly increasing in H, converge to zero as $H \to 0$ and to infinity as $H \to \infty$. We further observe that, as shown in the proof of Proposition 3, the terms π_1 and π_3 only depend on the ratio K/H. This term is uniquely pinned down by condition (32), which is the same for both economies. Therefore, $\pi_1(H, K, H)$ and $\pi_3(H, K, H)$ do not change as H changes.

Consider first the case with zero start-up cost, that is, $\tau = 0$. If $f_1(H) \leq 1$, as postulated in the proposition, the term $\beta f_1(H) < 1$. Because $\pi_3(H, K, H) < 0$ and $\beta f_1(H) < 1$, the right-hand side of (74) is bigger than the right-hand side of (75) for a given H. This implies that the value of H in the first equation must be bigger than in the second, that is, $H^{NC} > H^C$. Without capital obsolescence, $\pi_3(H, K, H) = 0$, and therefore, (74) and (75) are indistinguishable if $\tau = 0$.

Let us now consider the case $\tau > 0$. This variable only affects condition (74). Because $\beta f_1(H) < 1$, an increase in τ reduces the right-hand side of (74), which requires a lower value of H. For a sufficiently large τ, the steady-state level of knowledge declines to the point in which $H^{NC} < H^C$. *Q.E.D.*

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