Calculating Equivalent and Compensating Variations in CGE Models

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Introduction

Most CGE modelers use these models to assess the impacts of given shocks or policies on a specific economy. While it is quite straightforward to measure impacts on aggregate nominal production and consumption levels, relative prices, nominal income and savings, it is less obvious to **quantitatively** evaluate how much better or worse off the households are. As direct and indirect utility functions are purely ordinal in nature, we can only analyse the direction of change. An interesting alternative is provided by using the *money metric utility function*, which measures the nominal income the consumer needs at one set of prices in order to be as well of at an alternative set of prices and nominal income. As such, it can be used to obtain monetary measures of the welfare effects of different policy scenarios. The most common of these measures are equivalent variations (EV) and compensating variations (CV), which we will present in this note.

We might also be tempted to EVs and CVs between households. However, this is misleading as the utility each consumer receives from a given amount of income differs. In the same way, EVs and CVs should not, in principle, be aggregated across households as a welfare measure. Nonetheless, aggregate EVs and CVs could be interpreted as the total lump-sum transfer that is equivalent to the policy change that is examined. Note, finally, that even "disaggregate" EVs and CVs in CGE models are evaluated at the semi-aggregate level of household categories rather than at the individual consumer level. This brings us into the larger debate of aggregation in CGE models and the representative agent assumption that goes beyond this note (see Kirman 1992).

The objectives of the present memo is to provide MIMAP teams with a concise presentation of money metric welfare indicators and their calculation using the most popular utility functions found in CGE models. The general formulation will be presented as well as its derivation for the Cobb-Douglas and the LES utility functions.

Theory

Let us first define the *utility function*, $\mu(C)$, the *indirect utility function*, $V(P,Y)$ and the *money metric indirect utility function*, $m(P,\mu)$. In these expressions, C represents the vector of goods consumed, P the vector of prices and Y household income. Equation (1A) represents the utility function for a Cobb-Douglas specification and equation (1B) for the LES specification:

$$\mu^{CD}(C) = \prod_{i=1}^{I} C_i^{\alpha_i} \qquad (1A)$$

$$\mu^{LES}(C) = \prod_{i=1}^{I} (C_i - \gamma_i)^{\beta_i} \qquad (1B)$$

where $\sum_{i=1}^{I}\alpha_i = 1$ and $\sum_{i=1}^{I}\beta_i = 1$ and γ_i represents the minimum consumption level.

Given these specifications, demand functions are derived by maximizing utility subject to the budget constraint:

$$C_i^{CD}(P,Y) = \frac{\alpha_i Y}{P_i} \quad (2A)$$

$$C_i^{LES}(P,Y) = \gamma_i + \frac{\beta_i}{P_i}\left(Y - \sum_{i=1}^{I}\gamma_i P_i\right) \quad (2B)$$

The *indirect utility function, $V(P,Y)$,* is obtained by replacing the C_i in the utility functions (1A) and (1B) with the demand functions (2A) and (2B), respectively:

$$v^{CD}(P,Y) = \prod_{i=1}^{I}\left(\frac{\alpha_i Y}{P_i}\right)^{\alpha_i} \quad (3A)$$

$$\begin{aligned} v^{LES}(P,Y) &= \prod_{i=1}^{I}\left[\frac{\beta_i}{P_i}\left(Y - \sum_{i=1}^{I}\gamma_i P_i\right)\right]^{\beta_i} \\ &= \prod_{i=1}^{I}\left(\frac{\beta_i}{P_i}\right)^{\beta_i}\prod_{i=1}^{I}\left(Y - \sum_{i=1}^{I}\gamma_i P_i\right)^{\beta_i} \quad (3B) \\ &= \left(Y - \sum_{i=1}^{I}\gamma_i P_i\right)\prod_{i=1}^{I}\left(\frac{\beta_i}{P_i}\right)^{\beta_i} \end{aligned}$$

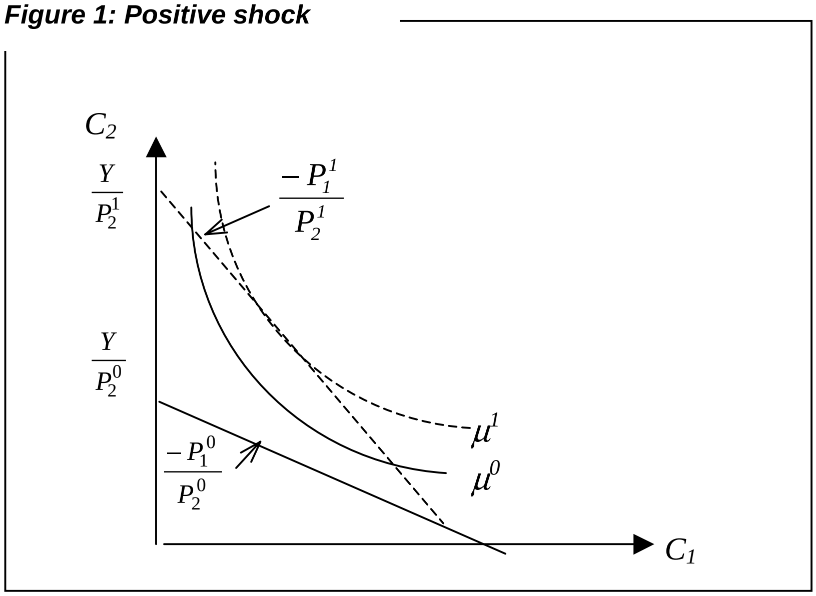
Solving equation (3A) and (3B) for Y gives the *money metric indirect utility function, $m(P,V)$,* which is a measure of the income needed to attain utility level v at the vector of prices P:

$$m^{CD}(P,v) = \prod_{i=1}^{I}\left(\frac{P_i}{\alpha_i}\right)^{\alpha_i} v \quad (4A)$$

$$m^{LES}(P,v) = \prod_{i=1}^{I}\left(\frac{P_i}{\beta_i}\right)^{\beta_i} v + \sum_{i=1}^{I}\gamma_i P_i \quad (4B)$$

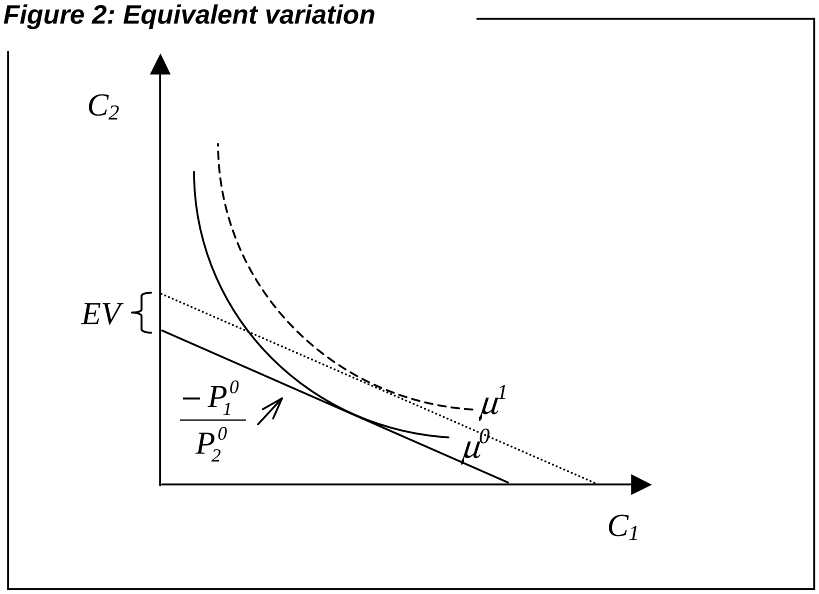
Equivalent and compensating variations are welfare measures based on the money metric indirect utility function. Before deriving the mathematical expressions for these measures, let us quickly recall graphically what they represent using a two-good example. Suppose that initially prices were (P_1^0, P_2^0). After some policy change, new prices (P_1^1, P_2^1) are observed assuming, for simplicity, that nominal income (Y) is unchanged. Figure 1 shows that, in this example, a higher utility level is attained after the policy change, since μ^1 is further than μ^0 from the origin.

Figure 1: Positive shock



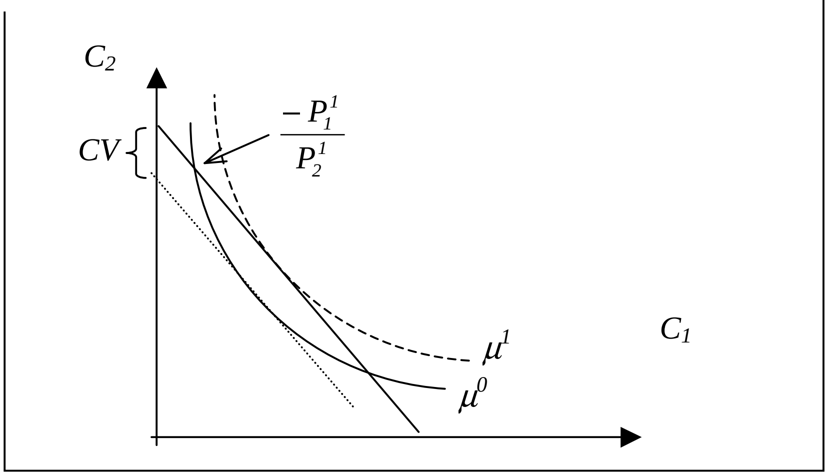
Although we can simply say that the policy had a positive impact on the consumer, we may want to quantitatively estimate the "size" of the impact in order to be able to compare between consumers and/or scenarios. The money metric indirect utility function allows such an evaluation. This is done by measuring the distance between the two indifference curves, at constant prices. Those prices could be the initial ones or the final ones. Figure 2 shows the first measure, which is called equivalent variation. The intuition behind that measure is to calculate what would be the income change, at initial prices, that is welfare-equivalent to the observed change in prices.

Figure 2: Equivalent variation



The second measure is known as the compensating variation (CV) and uses the post-policy prices. The CV thus measures the income change necessary to compensate the consumer for the change in prices. Figure 3 represents that measure.

Figure 3: Compensating variation



Let us now derive these two measures formally in the case where both prices and nominal income change. The equivalent variation is given by:

$$EV = m(P_i^0, v(P_i^1, Y^1)) - m(P_i^0, v(P_i^0, Y^0)) = m(P_i^0, v(P_i^1, Y^1)) - Y^0$$

For the Cobb-Douglas specification, we use equation (3A) and (4A):

$$\begin{aligned} EV &= m^{CD}(P_i^0, v^{CD}(P_i^1, Y^1)) - Y^0 \\ &= \prod_{i=1}^{I}\left(\frac{P_i^0}{\alpha_i}\right)^{\alpha_i} v^{CD}(P_i^1, Y^1) - Y^0 \\ &= \prod_{i=1}^{I}\left(\frac{P_i^0}{\alpha_i}\right)^{\alpha_i} \prod_{i=1}^{I}\left(\frac{\alpha_i Y^1}{P_i^1}\right)^{\alpha_i} - Y^0 \qquad (5A) \\ &= \prod_{i=1}^{I}\left(\frac{P_i^0}{P_i^1}\right)^{\alpha_i} Y^1 - Y^0 \end{aligned}$$

Similarly, for the LES specification, we use equation (3B) and (4B):

$$\begin{aligned} EV &= m^{LES}(P_i^0, v^{LES}(P_i^1, Y^1)) - Y^0 \\ &= \prod_{i=1}^{I}\left(\frac{P_i^0}{\beta_i}\right)^{\beta_i} v^{LES}(P_i^1, Y^1) + \sum_{i=1}^{I}\gamma_i P_i^0 - Y^0 \\ &= \prod_{i=1}^{I}\left(\frac{P_i^0}{\beta_i}\right)^{\beta_i}\left(Y^1 - \sum_{i=1}^{I}\gamma_i P_i^1\right)\prod_{i=1}^{I}\left(\frac{\beta_i}{P_i^1}\right)^{\beta_i} + \sum_{i=1}^{I}\gamma_i P_i^0 - Y^0 \qquad (5B) \\ &= \prod_{i=1}^{I}\left(\frac{P_i^0}{P_i^1}\right)^{\beta_i}\left(Y^1 - \sum_{i=1}^{I}\gamma_i P_i^1\right) - \left(Y^0 - \sum_{i=1}^{I}\gamma_i P_i^0\right) \end{aligned}$$

The compensating variation is given by the following expression:

$$CV = m(P_i^1, v(P_i^1, Y^1)) - m(P_i^1, v(P_i^0, Y^0)) = Y^1 - m(P_i^1, v(P_i^0, Y^0))$$

For the Cobb-Douglas specification and using equations (3A) and (4A):

$$\begin{aligned} CV &= Y^1 - m^{CD}(P_i^1, v^{CD}(P_i^0, Y^0)) \\ &= Y^1 - \prod_{i=1}^{I} \left(\frac{P_i^1}{\alpha_i} \right)^{\alpha_i} v^{CD}(P_i^0, Y^0) \\ &= Y^1 - \prod_{i=1}^{I} \left(\frac{P_i^1}{\alpha_i} \right)^{\alpha_i} \prod_{i=1}^{I} \left(\frac{\alpha_i Y^0}{P_i^0} \right)^{\alpha_i} \\ &= Y^1 - \prod_{i=1}^{I} \left(\frac{P_i^1}{P_i^0} \right)^{\alpha_i} Y^0 \end{aligned} \tag{6A}$$

And finally, for the LES specification, using equations (3B) and (4B):

$$\begin{aligned} CV &= Y^1 - m^{LES}(P_i^1, v^{LES}(P_i^0, Y^0)) \\ &= Y^1 - \prod_{i=1}^{I} \left(\frac{P_i^1}{\beta_i} \right)^{\beta_i} v^{LES}(P_i^0, Y^0) - \sum_{i=1}^{I} \gamma_i P_i^1 \\ &= \left(Y^1 - \sum_{i=1}^{I} \gamma_i P_i^1 \right) - \prod_{i=1}^{I} \left(\frac{P_i^1}{\beta_i} \right)^{\beta_i} \left(Y^0 - \sum_{i=1}^{I} \gamma_i P_i^0 \right) \prod_{i=1}^{I} \left(\frac{\beta_i}{P_i^0} \right)^{\beta_i} \\ &= \left(Y^1 - \sum_{i=1}^{I} \gamma_i P_i^1 \right) - \prod_{i=1}^{I} \left(\frac{P_i^1}{P_i^0} \right)^{\beta_i} \left(Y^0 - \sum_{i=1}^{I} \gamma_i P_i^0 \right) \end{aligned} \tag{6B}$$

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Kirman, A. (1992), "Whom or What does the Representative Individual Represent?", *Journal of Economic Perspectives*, 6(2): 117-136.

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