

**P**olitiques **E**conomiques et **P**auvreté  
**P**overty and **E**conomic **P**olicy

## **MODELING LABOR MARKETS IN CGE MODELS**

### **ENDOGENOUS LABOR SUPPLY, UNIONS AND EFFICIENCY WAGES**

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**May 2003**

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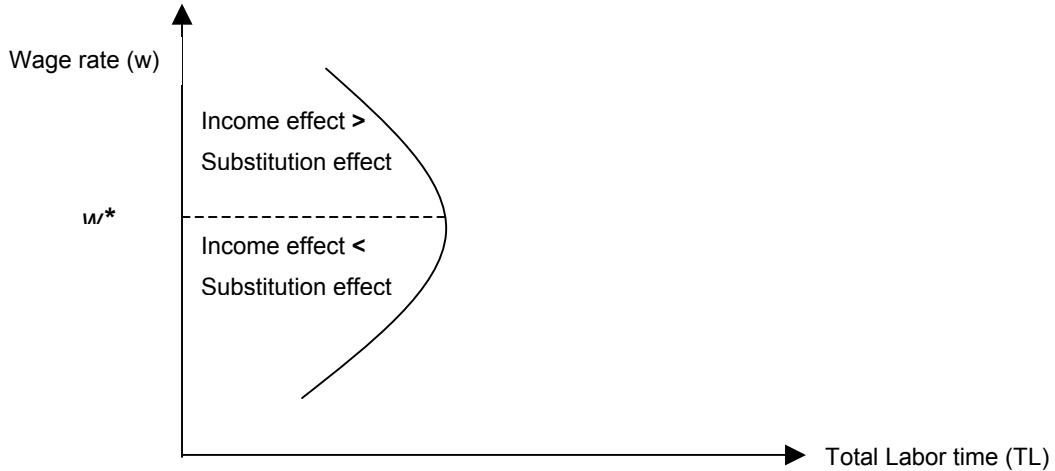
The aim of this paper is to explain how to introduce endogenous labor supply, labor market imperfections and non-Keynesian unemployment in a computable general equilibrium (CGE) model. For this purpose we present six models incorporating, respectively:

- Endogenous labor supply;
- Exogenous wage differentials;
- Wage bargaining and endogenous wage differential;
- Efficiency wage and full employment;
- Efficiency wage and unemployment;
- Endogenous labor supply and unemployment.

The remainder of this paper is as follows. In the first section, we introduce leisure into the utility function and determine the resulting demand functions for goods and leisure. We then transform use leisure demand to derive the labor supply function and present the corresponding calibration procedure. The second section deals with labor market imperfections. The first non-competitive case is characterized by the presence of unions and wage bargaining in the industrial sector. We introduce exogenous and endogenous wage differentials and explain the corresponding calibration procedure. The second source of imperfection is the efficiency wage which, in this case, is determined according to an effort incentive mechanism with imperfect monitoring. We construct two efficiency wage models with and without unemployment. We then combine the first and last models to include involuntary unemployment and endogenous labor supply simultaneously. In a final section, we contrast some key results with each of these models using our EXTER+ training model.

## **I. ENDOGENOUS LABOR SUPPLY**

In order to introduce endogenous labor supply, we assume that leisure is a normal good with an opportunity cost equal to the wage rate. An increase in the wage rate has income and substitution effects. On the one hand, the increase in the wage rate raises the opportunity cost of leisure and incites the consumer to work more (take less leisure). This is the substitution effect. On the other hand, the increase in the wage rate raises real income, which increases the consumption of all normal goods, including leisure. This is the income effect. The total effect on labor supply (leisure demand) is depicted in the next figure:



**Figure 2a:** The backward-bending labor supply curve<sup>1</sup>

When the income effect is less than the substitution effect, the consumer reacts to a rise in wage rates by reducing leisure and increasing labor time ( $w < w^*$ ). In the opposite situation, the labor supply curve has a negative slope ( $w > w^*$ ). The curve is thus said to be “Backward-bending” (Hanoch, 1965).

**I.1 Non labor income and leisure**

We consider a nested consumption function. At the first level, the consumer chooses between the consumption of an aggregate consumption good (C) and leisure time (TNL) according to a linear expenditure system (LES)<sup>2</sup>. At the second level, aggregate consumption is a Cobb-Douglas of the different consumption goods subject to a minimum consumption level ( $C^{min}$ ). If we consider the first level, the consumer program is the following:

$$\begin{aligned}
 \text{Max } U &= \alpha \ln(C - C^{min}) + \beta \ln TNL \\
 \text{s.c } pC &= wTL + y \\
 &= w(T - TNL) + y \\
 &= Y - wTNL
 \end{aligned}$$

where  $TL$  is labor time,  $T (=TNL+TL)$  is maximal disposal time,  $Y$  is Full income ( $=wT+y$ ),  $\alpha + \beta = 1$  and  $y$  is non labor income (benefits, transfers...).

Demand functions of consumption and leisure are:

<sup>1</sup>Hanosh (1965, p.639), Barzel and McDonald (1973, p.625) and Stern (1986) present the different possible shapes of the labor supply curve.

<sup>2</sup> The utility function is in fact an extended LES (Lluch 1973) where the variable “saving” is replaced by “leisure”.

$$C = C^{\min} + \frac{\alpha}{p}(Y - pC^{\min})$$

$$TNL = \frac{\beta}{w}(Y - pC^{\min})$$

With  $TL = T - TNL$ , the labor supply function is:

$$TL = T - \frac{\beta}{w}(Y - pC^{\min}) = \alpha T - \frac{\beta}{w}(y - pC^{\min})$$

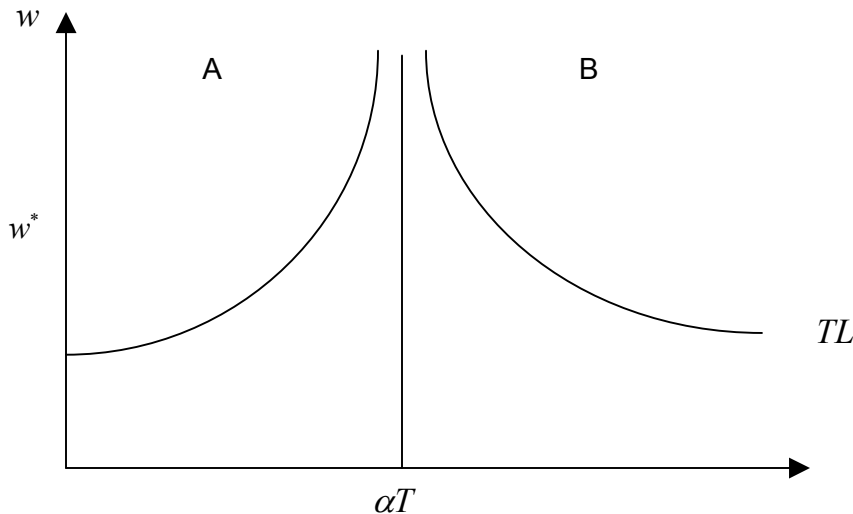
Let us first examine the impact of a change in non-labor income:

$$\frac{\partial TL}{\partial y} = -\frac{\beta}{w} < 0$$

We see that labor supply decreases through a pure income effect. As for the impact of a change in the wage rate, we obtain:

$$\frac{\partial TL}{\partial w} \frac{w}{TL} = \frac{\beta}{wTL}(y - pC^{\min})$$

which represents the wage rate elasticity of labor supply. It is positive if  $y > pC^{\min}$  and negative if  $y < pC^{\min}$ . If  $y = pC^{\min}$ , the labor supply is inelastic with respect to the wage rate and equals  $\alpha T$ . The value of this elasticity determines the shape of the labor supply curve that is represented in the following figure.



**Figure 2a:** The labor supply curve with non-labor income and minimal consumption<sup>3</sup>

<sup>3</sup>Hanosh (1965, p.639), Barzel and McDonald (1973, p.625) and Stern (1986) present the different possible shapes of the labor supply curve.

Note that the Labor supply curve A represents the case where  $y > pC^{\min}$  and is comparable to the bottom part of Figure 1 above. In this case, if  $TL = 0$ , we get  $w^* = \frac{y - pC^{\min}}{T} \cdot \frac{\beta}{\alpha}$ . However, in the case of an individual with a minimal consumption level superior to non-labor income ( $y < pC^{\min}$ ), we obtain an alternative, and quite different, shape represented by labor supply curve B. Finally, the vertical line at  $\alpha T$  corresponds to the case of an individual with  $y = pC^{\min}$ .

## I.2 Minimal level of leisure

The analysis remains the same if we consider that the agent has to consume a minimum of leisure  $TNL_{\min}$  to survive<sup>4</sup>. In this case the utility maximization program becomes:

$$\begin{aligned} \text{Max} U &= \alpha \ln(C - C^{\min}) + \beta \ln(TNL - TNL^{\min}) \\ \text{s.c} \quad & pC = Y - wTNL \end{aligned}$$

Leisure and good demand functions are:

$$\begin{aligned} C &= C^{\min} + \frac{\alpha}{p}(Y - wTNL^{\min} - pC^{\min}) \\ TNL &= TNL^{\min} + \frac{\beta}{w}(Y - wTNL^{\min} - pC^{\min}) \end{aligned}$$

The labor supply function ( $TL = T - TNL$ ) is as follows:

$$TL = T - TNL^{\min} - \frac{\beta}{w}(w(T - TNL^{\min}) + y - pC^{\min})$$

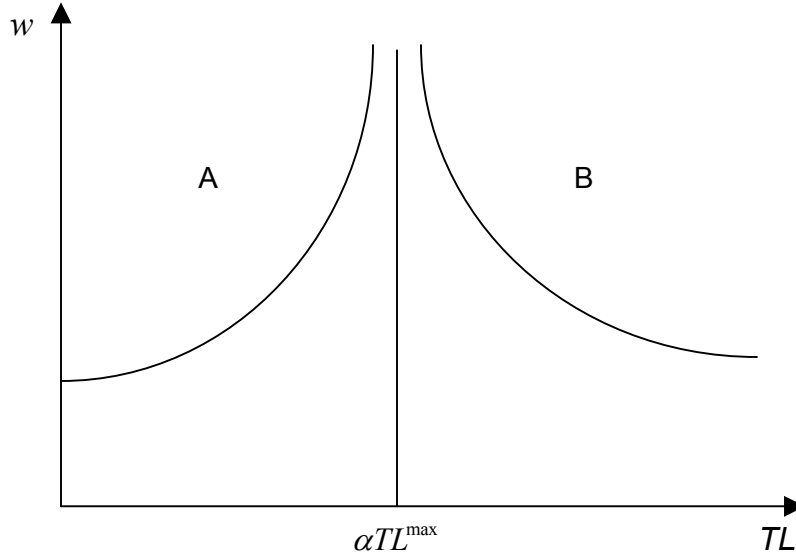
Defining  $TL^{\max}$  ( $= T - TNL^{\min}$ ), the maximal disposable level of work time, we obtain:

$$TL = \alpha TL^{\max} - \frac{\beta}{w}(y - pC^{\min})$$

In this case, the asymptote is represented by  $\alpha TL^{\max}$ .

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<sup>4</sup> Tarr (1989) p3-41, De Melo and Tarr(1992), Blonigen and al. (1997), Deaton and Muellbauer (1980) p95 et Berg and Reinert (1995) take account of the minimal level of leisure in the utility function.



**Figure 2b:** The labor supply curve with minimal leisure

### I.3 The EXTER+L model

We now integrate the results obtained in the previous section into the basic EXTER+ model in order to endogenize labor supply. Replacing  $y (= pC - wTL)$  in the labor supply function, we obtain:

$$\begin{aligned} TL &= \alpha TL^{\max} - \frac{\beta}{w} (pC - pC^{\min} - wTL) \\ &= TL^{\max} - \frac{\beta}{(1-\beta)w} (pC - pC^{\min}) \end{aligned}$$

Given full income  $Y = p.C + wTNL$ , we can rewrite the consumption demand function as:

$$pC = pC^{\min} + \frac{\alpha w}{\beta} (TNL - TNL^{\min})$$

Given  $T = TL + TNL$  and  $T = TL^{\max} + TNL^{\min}$ , we have:

$$\begin{aligned} TNL - TNL^{\min} &= TL^{\max} - TL \\ &= \frac{\beta}{(1-\beta)w} (pC - pC^{\min}) \quad (\text{using the TL expression above}) \end{aligned}$$

Replacing this, we obtain<sup>5</sup>:

$$pC = pC^{\min} + \frac{\alpha}{1-\beta} (pC - pC^{\min})$$

<sup>5</sup> It is clear that this expression simplifies to  $pC=pC$  when there is only one consumer good but, as we will see below, this is not the case when there are multiple consumer goods.

To translate this in terms of the EXTER+ model, we can now generalize the utility function to include multiple consumption goods<sup>6</sup>:

$$U = \sum \gamma_{tr} \ln(C_{tr} - C_{tr}^{\min}) + \beta \ln(TNL - TNL^{\min})$$

$$\text{with } \sum \gamma_{tr} + \beta = 1$$

We derive the following equations for the demand of goods and the labor supply (see appendix for the details of the model):

$$PC_{tr} C_{tr,h} = PC_{tr} C_{tr,h}^{\min} + \frac{\gamma_{tr,h}}{(1-\beta)} \left( CT_h - \sum_{trj} PC_{trj} C_{trj,h}^{\min} \right) \quad (1)$$

$$LS = Maxhour - \frac{\beta}{(1-\beta)w} \left( \sum_h CT_h - \sum_h \sum_{trj} PC_{trj} C_{trj,h}^{\min} \right) \quad (2)$$

Where  $Maxhour$  equals  $TL^{\max}$ ,  $LS$  equals  $TL$  and  $CT_h$  equals  $\sum_{tr} PC_{tr} C_{tr,h}$ . Note that

the expressions of the equivalent and the compensatory variations must be modified with respect to the EXTER+ model (See appendix).

Calibration procedures are explained in detail in what follows<sup>7</sup>. Income elasticities are adjusted proportionately in order to respect the Engel Aggregation: The income elasticity-weighted sum of individual consumptions divided by total consumption must equal one.

$$\left( \sum_{tr} \varepsilon_{tr,h} PC_{tr} C_{tr,h} \right) / CT_h = 1, \text{ with } \varepsilon_{tr,h} \text{ the income elasticity of demand.}$$

The share of leisure  $\beta$  is calibrated from the income elasticity of labor supply<sup>8</sup>:

$$\varepsilon_{ls} = - \frac{\beta CT_h}{(1-\beta)wLS}$$

Consumption shares  $\gamma_{tr,h}$  are calibrated from the adjusted income elasticity of demand:

$$\varepsilon_{tr,h} = \frac{\gamma_{tr,h} CT_h}{PC_{tr} C_{tr,h} (1-\beta)}$$

We use the **Frisch parameter**, defined as the negative value of the ratio of the disposal income (or total consumption) to supernumerary income:

<sup>6</sup> Tarr (1989) and De Melo and Tarr (1992).

<sup>7</sup> See Box 1 for the GAMS calibration procedure.

<sup>8</sup> The elasticity of labor supply with respect to the wage rate is:  $\varepsilon_{lw} = \left[ \frac{(1-\beta)Maxhour}{LS} \right]_{-1}$

$$Frisch_h = - \frac{CT_h}{CT_h - \sum_{trj} PC_{trj} C_{trj,h}^{\min}}$$

to derive the supernumerary income (consumption) that is used to calibrate the minimum consumption of each good from the demand function. The last step in the calibration procedure is the estimation of *Maxhour* from equation (2) that should be added to the model equations to endogenize labor supply *LS*<sup>9</sup>.

### Box 1: GAMS Calibration Procedure

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* Calibration Procedure for LES parameters
* Adjustment of the elasticities in order to respect Engel aggregation

CTHO (H) = YDHO (H) -SHO (H) ;
YELAS (TR, H) = YELAS (TR, H) / (SUM (TRJ, YELAS (TRJ, H) * PCO (TRJ) * CO (TRJ, H)) / CTHO (H)) ;

* Calibration of leisure share (Beta) from the income elasticity of labor
* supply (LELAS) and the consumption shares (Gamma) from the income
* elasticity of demand (YLEAS)
* Calibration of C_min with the Frisch parameter

LELAS = 0.12 ;
Beta = LSO * wo * LELAS / SUM (H, CTHO (H)) ;
Beta = (Beta / (1 + Beta)) ;
gamma (TR, H) = (PCO (TR) * CO (TR, H) * YELAS (TR, H) * (1 - Beta)) / CTHO (H) ;
V_MIN (H) = (SUM (TR, PCO (TR) * CO (TR, H))) * (1 + 1 / FRISCH (H)) ;
C_MIN (TR, H) = CO (TR, H) - (gamma (TR, H) / (1 - Beta)) * (CTHO (H) - V_MIN (H)) / PCO (TR) ;
MAXHOUR = LSO + (Beta / ((1 - Beta) * wo)) * SUM (H, (CTHO (H) - V_MIN (H))) ;

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## II. LABOR MARKET IMPERFECTIONS

Efficiency wage and union bargaining theories explain how wages are determined and labor market equilibrium is attained in a non-competitive context. They are based in an intertemporal framework where labor moves between employment and unemployment.

Efficiency wage theory explains the generally observed positive relationship between the wage rate and worker productivity. We find four underlying explanations in the literature. The first is based on effort incentive mechanism with imperfect monitoring (Shapiro and Stiglitz 1984). The second assumes that the firm fixes high wages in order to attract the most productive workers. The third takes into account hiring and training costs, which lead firms to try to retain its workers through higher wages. Finally, in the sociological approach, the worker

<sup>9</sup> Ballard and al. (1984) fix *Maxhour* arbitrarily and use this, rather than the income elasticity of labor supply, to calibrate the share of leisure.



considers that his remuneration reflects the equity of wage relationship. Akerlof and Yellen (1986) and Cahuc and Zylberberg (1996) present these different explanations in detail. In the present case we restrict our application to the effort incentive mechanism with imperfect monitoring.

Union theory analyzes wage bargaining processes. The union's objective is to maximize employment and wage levels. It may be assumed that the union maximizes either total labor remuneration or worker utility. The union is deemed passive when it bargains only salaries. In this case it maximizes its utility under the firm labor demand constraint (De Melo et Tarr 1992). The union is deemed active when it bargains both wage and employment level (Mc Donald and Solow 1981 and Devarajan and al. 1997). In this case the labor market equilibrium is the tangency between the union indifference curve and the firm isoprofit curve. Fisher and Waschik (2000) use an axiomatic approach of wage bargaining maximizing Nash criterion. This latter equals the weighed firm's profit multiplied by a modified Stone-Geary utility function of the union. In this paper, we study the case of a passive union<sup>10</sup>.

In section II.1, we present a CGE model with wage differentials due to the presence of unions. Section II.2 presents two efficiency wage models. The first applied model supposes full employment and the second, involuntary unemployment. Finally, in section II.3, we present an application with endogenous labor supply and efficiency wage.

## II.1. Union bargaining: EXTER+U model

The objective of the passive union is to maximize its utility function under the labor demand constraint<sup>11</sup>:

$$\begin{aligned} \text{Max } U &= (w\Phi - w^{\min})^{\mu} \left( LD - \overline{LD}^{\min} \right)^{1-\mu} \\ \text{s.c } LD &= \left( \frac{\alpha}{1-\alpha} \right)^{\sigma} \left( \frac{r}{w\Phi} \right)^{\sigma} KD \end{aligned}$$

where  $w, \Phi, w^{\min}, LD, \overline{LD}^{\min}, KD, r, \alpha, \sigma$  and  $\mu$  are, respectively, the non-unionized wage rate, the union wage differential, the minimum wage rate acceptable to the union, labor demand, the fixed minimum level of employment acceptable to the union, capital demand, returns to capital,

<sup>10</sup> Farber (1986) presents a survey of the union and wage bargaining theory.

<sup>11</sup> The Stone-Geary utility function was first used in the union framework by Detrouzos and Pencavel (1981).

share parameter in the value added CES function, the elasticity of substitution between capital and labor, and the weight given by the union to the supernumerary wage.

If we replace the labor demand expression in the utility function, assume that  $w^{min}$  is equal to  $w$ , and maximize utility with respect to  $\Phi$  we obtain the following expression:

$$\frac{(\Phi - 1)}{\Phi} = \frac{\mu}{(1 - \mu)\sigma} \frac{(LD - \overline{LD}^{min})}{LD} \tag{3}$$

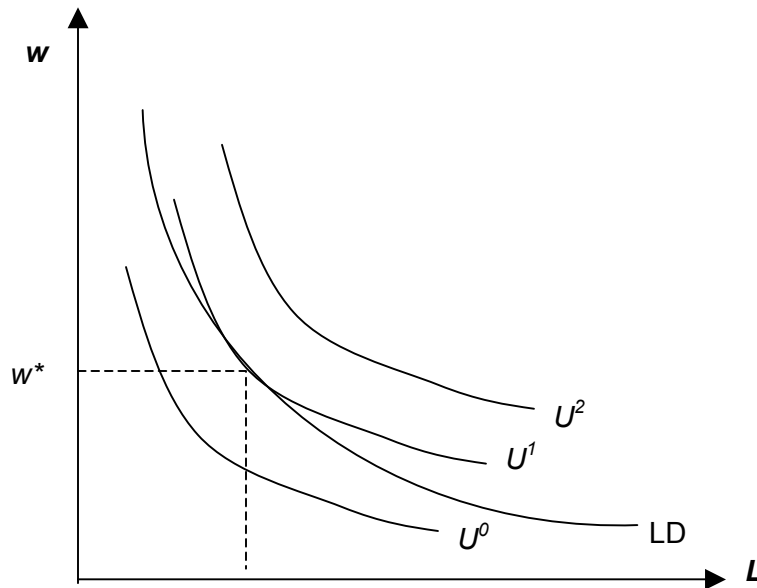
This equilibrium is represented by the tangency between the union indifference curve and the labor demand function in Figure 4 below.

To introduce equation (3) in the EXTER+ model, we first fix the wage differential and the weight given to the supernumerary wage, and we calibrate the minimum level of employment. The wage differential is then used in the calibration of the labor demand volume. It is also used in the equation of household income where we replace the wage rate  $w$  by  $w\Phi$ :

$$YH = \lambda_w w \Phi \sum LD + \lambda_r \sum rKD + \dots$$

Lastly, the wage differential is used in the calibration of the labor income share  $\lambda_w$  and in the residual return to capital equation:

$$r = \frac{P_v VA - w \Phi LD}{KD}$$



**Figure 3:** Unions and Wage Bargaining.

## II.2. Efficiency wage

### II.2.1. Efficiency wage and full employment: EXTER+E model

In what follows, we present a modified version of an efficiency wage model without unemployment (**Thierfelder and Shiells, 1997**). We assume that industrial firms pay a wage premium. In this efficiency wage sector, the representative worker has the possibility to shirk and to increase his utility by a fraction  $u$ . His expected utility is:

$$rU_s = w\Phi(1+u) - (q+b)(U_s - U_c) \quad (4)$$

where  $r$  is the discount rate<sup>12</sup>,  $U^s$  is the utility of a shirker,  $u$  is the added utility from shirking,  $q$  is the exogenous probability that a worker will lose his/her job in the efficiency wage sector (and be forced to take a job in the lesser-paying competitive sector),  $b$  is the probability of getting caught shirking and  $U_c$  is the utility of a worker in the competitive sector.

When a worker does not shirk ( $n$ ) he does not get the added utility  $u$ , but he is less likely to leave the efficiency wage sector. His expected utility is:

$$rU_n = w\Phi - (q+p_1)(U_n - U_c) \quad (5)$$

where  $U_n$  is the utility of a non-shirker and  $p_1$  is the probability of being wrongly accused of shirking (and thus losing one's job).

In equilibrium, employers in the efficiency wage sector ensure that the wage differential is sufficient to ensure that workers do not shirk:

$$U_n - U_s \geq 0$$

Thus there are  $(q+p_1)L$  workers in the efficiency wage sector who lose their jobs in equilibrium, where  $L$  is the total number of workers in this sector. Workers from the competitive sector replace fired workers in the efficiency wage sector with equal probability:

$$rU_c = w + \left( \frac{(q+p_1)L}{L_c} \right) (U_n - U_c) \quad (6)$$

where  $LS = L + L_c$ , represents the total supply of labor. Manipulating equation (4), (5) and (6) we obtain the following expression:

$$\frac{(\Phi-1)}{\Phi} = \frac{r u}{(b-p_1)} + \frac{u (p_1+q) LS}{(b-p_1) (LS - LD_{ind})} \quad (7)$$

Equation (7) is called the non-shirking condition. It is introduced in the model by calibrating the supplementary utility from shirking  $u$ .

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<sup>12</sup> We assume that wages are paid at the end of the period.

### II.2.2. Efficiency wage and unemployment: EXTER+C model

We present here an application of Shapiro and Stiglitz (1984) in a CGE framework. The expected utility of a shirking worker ( $U_s$ ):

$$rU_s = w - (q+b) (U_s - U_u) \quad (8)$$

where  $q$ ,  $b$ ,  $r$ ,  $w$  and  $U_u$  represent respectively the probability to be caught, the exogenous quit, discount and wage rates, and the utility of an unemployed worker. The expected utility of a non-shirker ( $U_n$ ):

$$rU_n = w - e - b (U_n - U_u) \quad (9)$$

where  $e \geq 0$  represents the disutility of effort. For a shirking worker,  $e$  equals zero. The utility for an unemployed worker is:

$$rU_u = \bar{w} + a (U_{eu} - U_u) \quad (10)$$

where  $\bar{w}$  : Unemployment benefits

$a$  : Probability to be hired

$U_{eu}$  : Expected utility for a worker equals to  $U_n$  in the equilibrium.

The worker decides to not shirk if  $U_n \geq U_s$ . This inequality determines the non-shirking condition (NSC). With equations (8) and (9), this condition can be rewritten as follows:

$$w \geq rU_u + \frac{e(r+b+q)}{q} \quad (11)$$

Assuming that in the equilibrium  $U_{eu} = U_n$  and equation (11) is a strict equality, we can use equation (10) to obtain:

$$w \geq \bar{w} + e + \frac{e(a+b+r)}{q}$$

From this equation we remark that the efficiency wage is increasing with respect to the exogenous quit rate  $b$ , the probability to be hired  $a$ , the discount rate  $r$ , the unemployment benefits  $\bar{w}$  and the disutility of effort  $e$ , and decreasing with respect to the probability to be caught, which reflects the monitoring capacity of the firm.

In equilibrium, the flow into the unemployment pool is equal to the flow out:

$$\overbrace{bL}^{\text{Flow into the pool}} = \overbrace{a(LS - L)}^{\text{Flow out of the pool}}$$

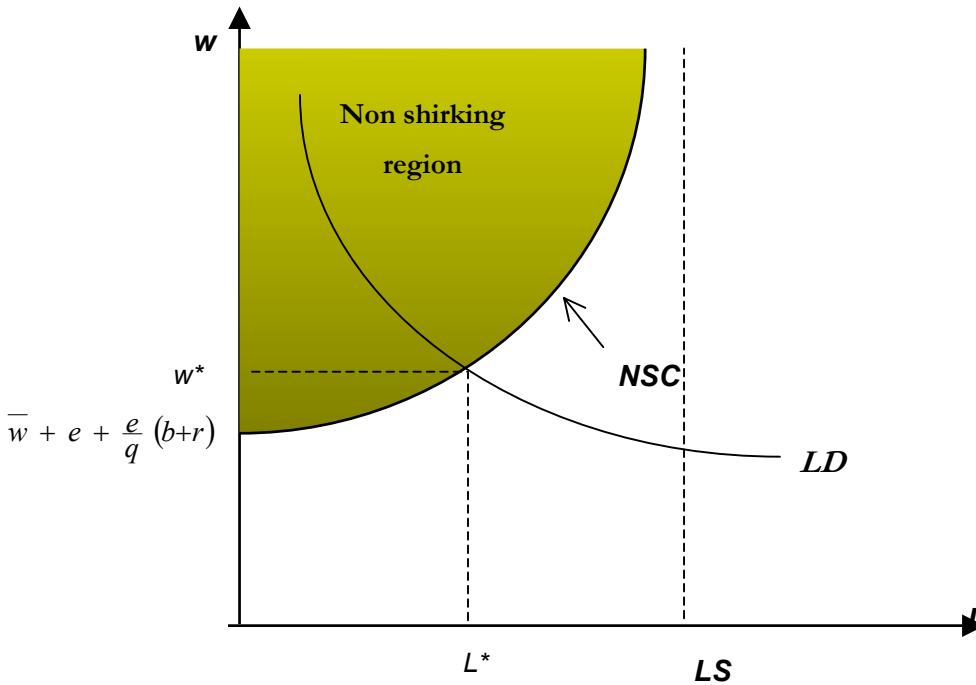
With the unemployment rate,

$$u = \frac{LS - L}{LS}$$

the non shirking condition becomes:

$$w = \bar{w} + e + \frac{e}{q} \left( \frac{b}{u} + r \right) \quad (\text{NSC})$$

This condition is depicted in Figure 4 where the equilibrium wage rate is negatively related with the unemployment rate. The labor market equilibrium is characterized by the presence of involuntary unemployment.



**Figure 4:** Involuntary unemployment equilibrium

The NSC is included in the model with the calibration of the parameter  $e$ .

### II.2.3. Unemployment and endogenous labor supply: EXTER+LC

In this model we combine the structure of two models: The model with endogenous labor supply (EXTER+L), and the model with efficiency wage and unemployment (EXTER+C).

### III. A COMPARISON OF RESULTS WITH LABOR MARKET MODELS

In the following table, we contrast the effects of a tariff reduction on the wage rate and labor supply in the EXTER+L model<sup>13</sup>. The introduction of endogenous labor supply implies that some of the adjustment to reduced labor demand is absorbed by a reduction in labor supply (increase in leisure demand), so that wage rates fall less than in the exogenous labor supply EXTER+ model. Substituting leisure for good consumption reduces the gains from tariff removal. The greater the elasticity of labor supply with respect to income the less the increase in good consumption relatively to leisure and the lower the benefits from liberalization<sup>14</sup>. These results are consistent with de Melo and Tarr (1992).

We also contrast the results of a tariff reduction on the wage rate and labor supply in the EXTER+U and EXTER+E models<sup>15</sup>. We first assume an exogenous wage differential without including equation (3) (EXTER+Ue) and an endogenous wage differential using different values for the weight of the supernumerary wage rate in the union decision. The welfare gains from liberalization are less when exogenous factor price distortions are present (EXTER+Ue). The resource allocation is inefficient when there is a wedge between factor price and marginal productivity. Welfare effects are therefore reduced.

The tariff removal reduces the endogenous wage distortions in the union model EXTER+U. Welfare effects are increasing with the union weight on wages  $\mu$ . The greater is  $\mu$  the less is the reduction in the wage rate. Household income decreases less than in the model with exogenous wage differential EXTER+Ue. Welfare effects are greater when the labor market distortion is endogenous rather than exogenous.

The results of liberalization with the EXTER+E model are quite similar to those obtained with EXTER+Ue model except for the endogenous wage differential and labor demand in the industrial sector which contracts less than when the wage distortion is exogenous.

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<sup>13</sup> We assume the following parameter values:  $\sigma = 1.5$ ,  $\Phi = 1.2$  and  $0 < \mu < 1$  and -0.12 for the elasticity of labor supply with respect to income.

<sup>14</sup> With an elasticity of labor supply equal to zero we get the results of the basic model, EXTER+, with exogenous labor supply.

<sup>15</sup> We assume  $p_1 = 0.1$ ,  $p_2 = 0.3$ ,  $q = 0.5$ ,  $u$  is calibrated,  $r = 0.05$ ,  $\Phi = 1.2$ .

In the Efficiency wage case with unemployment, the values of  $b$ ,  $q$  and  $r$  are fixed respectively to 0.1, 0.3 and 0.05. Unemployment benefits  $\bar{w}$  are supposed equal to zero. We remark that the effects on unemployment are more important with EXTER+LC than EXTER+C. Recall the negative relation between the wage rate and unemployment in the efficiency wage as in the wage curve literature<sup>16</sup>. The tariff removal reduces wage rate and increases unemployment. The income effects are negative and greater than those obtained with the rest of the models. Hence the welfare effects are negative. When labor supply is endogenous (EXTER+LC) the wage rate decreases more than in the case with fixed supply and labor demand decreases less. To get the corresponding increase in the unemployment rate labor supply has to increase.

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<sup>16</sup> Blanchflower, D. G and Oswald, A. J. (1995).

**Table 1: Total tariff cut effects**

% Variation	EXTER+	Endogenous labor supply EXTER+L	Exogenous wage differential EXTER+Ue	Union bargaining EXTER+U			Efficiency wage EXTER+E	Efficiency wage EXTER+C	Efficiency wage Exter+LC	
Weight on wages	-	-	-	0.30	0.60	0.90	1.00	-	-	-
Wage rate	-5.41	-5.39	-5.32	-5.08	-4.41	-3.71	-3.45	-5.29	-5.10	-5.11
Labor demand (Industry)	-15.65	-15.71	-15.97	-13.99	-8.31	-2.28	-0.02	-15.77	-16.75	-16.71
Wage differential	-	-	0.00	-2.27	-8.31	-14.04	-16.02	-0.24	-	-
Labor supply	-	-0.04	-	-	-	-	-	-	0.00	0.04
Unemployment rate	-	-	-	-	-	-	-	-	7.39	7.41
Household income	-5.98	-6.00	-6.10	-6.07	-5.98	-5.90	-5.87	-6.10	-6.39	-6.38
Rural poor	-5.45	-5.48	-5.58	-5.50	-5.28	-5.07	-5.00	-5.58	-5.91	-5.89
Urban poor	-5.84	-5.87	-5.97	-5.90	-5.70	-5.50	-5.43	-5.96	-6.30	-6.28
Rural rich	-4.76	-4.78	-4.87	-4.83	-4.72	-4.61	-4.57	-4.87	-5.14	-5.13
Urban rich	-7.53	-7.54	-7.62	-7.70	-7.93	-8.17	-8.26	-7.63	-7.85	-7.83
Equivalent Variation	0.59	0.587	0.42	0.45	0.52	0.57	0.58	0.42	-0.02	-0.02
Rural poor	-1.06	-1.16	-1.24	-1.22	-1.19	-1.19	-1.19	-1.23	-1.68	-1.79
Urban poor	-0.74	-0.83	-0.91	-0.87	-0.77	-0.70	-0.67	-0.91	-1.34	-1.44
Rural rich	1.64	1.74	1.46	1.48	1.53	1.56	1.57	1.47	1.03	1.15
Urban rich	3.60	3.76	3.43	3.47	3.58	3.67	3.70	3.43	3.00	3.18

**Note:** Wage differential in the industrial sector is fixed in the benchmark to 1.2 ; the labor supply elasticity is -0.12 and the base run unemployment rate is 10%.



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## APPENDIX : EXTER + AND ENDOGENOUS LABOR SUPPLY

### Model structure

#### Production

1.  $XS_j = \min \left[ \frac{CI_j}{io_j}, \frac{VA_j}{v_j} \right]$  4
2.  $VA_{nag} = A_{nag}^{KL} \left[ \alpha_{nag}^{KL} LD_{nag}^{-\rho_{nag}^{KL}} + (1 - \alpha_{nag}^{KL}) KD_{nag}^{-\rho_{nag}^{KL}} \right]^{-1/\rho_{nag}^{KL}}$  2
3.  $VA_{AGR} = A_{tr}^{CL} \left[ \alpha^{CL} CF^{-\rho^{CL}} + (1 - \alpha^{CL}) LAND^{-\rho^{CL}} \right]^{-1/\rho^{CL}}$  1
4.  $CF = A_{AGR}^{KL} \left[ \alpha_{AGR}^{KL} LD_{AGR}^{-\rho_{AGR}^{KL}} + (1 - \alpha_{AGR}^{KL}) KD_{AGR}^{-\rho_{AGR}^{KL}} \right]^{-1/\rho_{AGR}^{KL}}$  1
5.  $VA_{ntr} = LD_{ntr}$  1
6.  $CI_j = io_j XS_j$  4
7.  $DI_{tr,j} = aij_{tr,j} CI_j$  12
8.  $LAND = \left( \frac{1 - \alpha^{CL}}{\alpha^{CL}} \right)^{\sigma^{CL}} \left( \frac{rc}{rl} \right)^{\sigma^{CL}} CF$  1
9.  $LD_{tr} = \left( \frac{\alpha_{tr}^{KL}}{1 - \alpha_{tr}^{KL}} \right)^{\sigma_{tr}^{KL}} \left( \frac{r_{tr}}{w} \right)^{\sigma_{tr}^{KL}} KD_{tr}$  3
10.  $LD_{NTR} = \frac{P_{NTR} XS_{NTR} - \sum_{tr} PD_{tr} DI_{tr,NTR}}{w}$  1

**Income and savings**

11. 
$$YH_h = \lambda_h^W \cdot w \sum_j LD_j + \lambda_h^R \sum_{tr} r_{tr} KD_{tr} + \lambda_h^L \cdot rl \cdot LAND + PINDEX \cdot TG_h$$
 4
12. 
$$YDH_h = YH_h - DTH_h$$
 4
13. 
$$SH_h = v \cdot \psi_h \cdot YDH_h$$
 4
14. 
$$YF = \lambda^{RF} \sum_{tr} r_{tr} KD_{tr} + \lambda^{LF} \cdot rl \cdot LAND$$
 1
15. 
$$SF = YF - \sum_h DIV_h - e \cdot DIV^{ROW} - DTF$$
 1
16. 
$$YG = \sum_{tr} TI_{tr} + \sum_{tr} TIE_{tr} + \sum_{tr} TIM_{tr} + \sum_h DTH_h + DTF$$
 1
17. 
$$SG = YG - G - PINDEX \sum_h TG_h$$
 1
18. 
$$TI_{tr} = tx_{tr} (P_{tr} XS_{tr} - PE_{tr} EX_{tr}) + tx_{tr} (1 + tm_{tr}) e PWM_{tr} M_{tr}$$
 3
19. 
$$TIM_{tr} = tm_{tr} e PWM_{tr} M_{tr}$$
 3
20. 
$$TIE_{tr} = te_{tr} PE_{tr} EX_{tr}$$
 3
21. 
$$DTH_h = tyh_h YH_h$$
 4
22. 
$$DTF = tyf \cdot YF$$
 1

**Demand**

23. 
$$CTH_h = YDH_h - SH_h$$
 4
- 24. 
$$PC_{tr} C_{tr,h} = PC_{tr} C_{tr,h}^{MIN} + \frac{\gamma_{tr,h}}{(1-\beta_l)} \left( CTH_h - \sum_{trj} PC_{trj} C_{trj,h}^{MIN} \right)$$
 12
25. 
$$G = XS_{ntr} P_{ntr}$$
 1
26. 
$$INV_{tr} = \frac{\mu_{tr} IT}{PC_{tr}}$$
 3

$$27. ITVOL \cdot PINV = IT \quad 1$$

$$28. DIT_{tr} = \sum_j DI_j \quad 3$$

### Prices

$$29. PV_j = \frac{P_j XS_j - \sum PC_{tr} DI_{tr,j}}{VA_j} \quad 4$$

$$30. r_{nag} = \frac{PV_{nag} VA_{nag} - w LD_{nag}}{KD_{nag}} \quad 2$$

$$31. r_{AGR} = \frac{rc \cdot CF - w LD_{AGR}}{KD_{AGR}} \quad 1$$

$$32. rc = \frac{PV_{AGR} VA_{AGR} - rl \cdot LAND}{CF} \quad 1$$

$$33. PD_{tr} = (1 + tx_{tr}) PL_{tr} \quad 3$$

$$34. PM_{tr} = (1 + tx_{tr}) (1 + tm_{tr}) e \cdot PWM_{tr} \quad 3$$

$$35. PE_{tr} = \frac{e \cdot PWE_{tr}}{1 + te_{tr}} \quad 3$$

$$36. PC_{tr} Q_{tr} = PD_{tr} D_{tr} + PM_{tr} M_{tr} \quad 3$$

$$37. P_{tr} XS_{tr} = PL_{tr} D_{tr} + PE_{tr} EX_{tr} \quad 3$$

$$38. PINV = \prod_{tr} \left( \frac{PC_{tr}}{\mu_{tr}} \right)^{\mu_{tr}} \quad 1$$

$$39. PINDEX = \sum_i \delta_i PV_i \quad 1$$

**International Trade**

$$40. XS_{tr} = B_{tr}^E \left[ \beta_{tr}^E EX_{tr}^{\kappa_{tr}^E} + (1 - \beta_{tr}^E) D_{tr}^{\kappa_{tr}^E} \right]^{\frac{1}{\kappa_{tr}^E}} \quad 3$$

$$41. EX_{tr} = \left[ \left( \frac{PE_{tr}}{PL_{tr}} \right) \left( \frac{1 - \beta_{tr}^E}{\beta_{tr}^E} \right) \right]^{\tau_{tr}^E} D_{tr} \quad 3$$

$$42. Q_{tr} = A_{tr}^M \left[ \alpha_{tr}^M M_{tr}^{-\rho_{tr}^M} + (1 - \alpha_{tr}^M) D_{tr}^{-\rho_{tr}^M} \right]^{\frac{-1}{\rho_{tr}^M}} \quad 3$$

$$43. M_{tr} = \left[ \left( \frac{PD_{tr}}{PM_{tr}} \right) \left( \frac{\alpha_{tr}^M}{1 - \alpha_{tr}^M} \right) \right]^{\sigma_{tr}^M} D_{tr} \quad 3$$

$$44. CAB = \sum_{tr} PWM_{tr} M_{tr} + \lambda^{ROW} \sum_{tr} r_{tr} KD_{tr} / e + \lambda^{LROW} rl \cdot LAND / e \\ + DIV^{ROW} - \sum_{tr} PWE_{tr} EX_{tr} \quad 1$$

**Equilibrium**

$$45. Q_{tr} = DIT_{tr} + \sum_h C_{tr,h} + INV_{tr} \quad 3$$

$$46. LS = \sum_j LD_j$$

$$\longrightarrow 47. LS = Maxhour - \frac{\beta_l}{w(1-\beta_l)} \left( \sum_h CTH_h - \sum_h \sum_{trj} PC_{trj} C_{trj,h}^{MIN} \right) \quad 1$$

$$48. IT = \sum_h SH_h + SF + SG + e \cdot CAB \quad 1$$

$$\longrightarrow 49. EV_h = \frac{1}{1-\beta_l} \left( \left( CTH_h - \sum_{trj} PC_{trj} C_{trj,rj}^{MIN} \right) \prod_{tr} \left[ \frac{PCO_{tr}}{PC_{tr}} \right]^{\gamma_{tr,h}} \cdot \left( \frac{w_0}{w} \right)^{\beta_l} \right. \\ \left. - \left( CTHO_h - \sum_{trj} PCO_{trj} C_{trj,rj}^{MIN} \right) \right) \quad 4$$

~~Total: 132~~**Endogenous variables**

		<b>Number of variables</b>
$C_{tr,h}$	Household $h$ 's consumption of good $tr$ (volume)	12
$CF$	Composite agricultural capital-labor factor (volume)	1
$CI_j$	Total intermediate consumption of activity $j$ (volume)	4
$CTH_h$	Household $h$ 's total consumption (value)	4
$D_{tr}$	Demand for domestic good $tr$ (volume)	3
$DI_{tr,j}$	Intermediate consumption of good $tr$ in activity $j$ (volume)	12
$DIT_{tr}$	Intermediate demand for good $tr$ (volume)	3
$DTF$	Receipts from direct taxation on firms' income	1
$DTH_h$	Receipts from direct taxation on household $h$ 's income	4
$e$	Exchange rate	1
$EV_h$	Equivalent variation for household $h$	4
$EX_{tr}$	Exports in good $tr$ (volume)	3
$G$	Public expenditures	1
$INV_{tr}$	Investment demand for good $tr$ (volume)	3
$IT$	Total investment	1
$LD_j$	Activity $j$ demand for labor (volume)	4
$M_{tr}$	Imports in good $tr$ (volume)	3
$v$	Adjustment variable for household's savings	1
$P_i$	Producer price of good $i$	4
$PC_{tr}$	Consumer price of composite good $tr$	3
$PD_{tr}$	Domestic price of good $tr$ including taxes	3

$PE_{tr}$ :	Domestic price of exported good $tr$	3
$PINDEX$ :	GDP deflator	1
$PINV$ :	Price index of investment	1
$PL_{tr}$ :	Domestic price of good $tr$ (excluding taxes)	3
$PM_{tr}$ :	Domestic price of imported good $tr$	3
$PV_j$ :	Value added price for activity $j$	4
$Q_{tr}$ :	Demand for composite good $tr$ (volume)	3
$r_{tr}$ :	Rate of return to capital in activity $tr$	3
$rl$ :	Rate of return to agricultural land	1
$rc$ :	Rate of return to composite factor	1
$SF$ :	Firms' savings	1
$SG$ :	Government's savings	1
$SH_h$ :	Household $h$ 's savings	4
$TI_{tr}$ :	Receipts from indirect tax on $tr$	3
$TIE_{tr}$ :	Receipts from tax on export $tr$	3
$TIM_{tr}$ :	Receipts from import duties $tr$	3
$VA_j$ :	Value added for activity $j$ (volume)	4
$w$ :	Wage rate	1
$XS_{tr}$ :	Output of activity $tr$ (volume)	3
$YDH_h$ :	Household $h$ 's disposable income	4
$YF$ :	Firms' income	1
$YG$ :	Government's income	1
$YH_h$ :	Household $h$ 's income	4
$LS$ :	Total labor supply (volume)	1





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**Total: 132**
**Exogenous variables**

	<b>Number of variables</b>	
$CAB$ :	Current account balance	1
$DIV_h$ :	Dividends paid to household $h$	4
$DIV^{ROW}$ :	Dividends paid to the rest of the World	1
$ITVOL$ :	Total investment (volume)	1
$KD_{tr}$ :	Demand for capital in activity $tr$ (volume)	3
$LAND$ :	Land supply (volume)	1
$PWE_{tr}$ :	World price of export $tr$	3
$PWM_{tr}$ :	World price of import $tr$	3
$TG_h$ :	Public transfers to household $h$	4
$XS_{NTR}$ :	Output of activity $NTR$ (volume)	1
		<hr/> <b>Total: 22</b>

**Parameters****Production functions**

$A_j$ :	Scale coefficient (Cobb-Douglas production function)
$aij_{tr,j}$ :	Input-output coefficient
$\alpha_j$ :	Elasticity (Cobb-Douglas production function)
$io_j$ :	Technical coefficient (Leontief production function)
$v_j$ :	Technical coefficient (Leontief production function)

### **CES function between capital and labor**

$A_{tr}^{KL}$  : Scale coefficient

$\alpha_{tr}^{KL}$  : Share parameter

$\rho_{tr}^{KL}$  : Substitution parameter

$\sigma_{tr}^{KL}$  : Substitution elasticity

### **CES function between composite factor and land**

$A_{tr}^{CL}$  : Scale coefficient

$\alpha_{tr}^{CL}$  : Share parameter

$\rho_{tr}^{CL}$  : Substitution parameter

$\sigma_{tr}^{CL}$  : Substitution elasticity

### **CES function between imports and domestic production**

$A_{tr}^M$  : Scale coefficient

$\alpha_{tr}^M$  : Share parameter

$\rho_{tr}^M$  : Substitution parameter

$\sigma_{tr}^M$  : Substitution elasticity

### **CET function between domestic production and exports**

$B_{tr}^E$  : Scale coefficient

$\beta_{tr}^E$  : Share parameter

$\kappa_{tr}^E$  : Transformation parameter

$\tau_{tr}^E$  : Transformation elasticity

### LES consumption function

$\gamma_{tr,h}$  : Marginal share of good  $tr$

$\beta_l$  : Marginal share of leisure

$Maxhour$  : Maximal time

$C_{tr,h}^{MIN}$  : Minimum consumption of good  $tr$

### Tax rates

$te_{tr}$  : Tax on exports  $tr$

$tm_{tr}$  : Import duties on good  $tr$

$tx_{tr}$  : Tax rate on good  $tr$

$tyh_h$  : Direct tax rate on household  $h$ 's income

$tyf$  : Direct tax rate on firms' income

### Other parameters

$\delta_j$  : Share of activity  $j$  in total value added

$\lambda_h^L$  : Share of land income received by household  $h$

$\lambda^{LF}$  : Share of land income received by firms

$\lambda^{LROW}$  : Share of land income received by foreigners

$\lambda_h^R$  : Share of capital income received by household  $h$

$\lambda^{RF}$  : Share of capital income received by firms

$\lambda^{ROW}$  : Share of capital income received by foreigners

$\lambda_h^W$  : Share of labor income received by household  $h$

$\psi_h$  : Propensity to save

$\mu_{tr}$  : Share of the value of good  $tr$  in total investment

**sets**

- $i, j \in I = \{AGR, IND, SER, NTR\}$  All activities and goods (*AGR*: agriculture, *IND*: industry, *SER*: services, *NTR*: non-tradable services)
- $tr \in TR = \{AGR, IND, SER\}$  Tradable activities and goods
- $nag \in NAG = \{IND, SER\}$  Non-agricultural Tradable activities and goods
- $h \in H = \{RP, UP, RR, UR\}$  Households (*RP*: rural poor, *UP*: urban poor, *RR*: rural rich, *UR*: urban rich)