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This paper shows how confirmatory factor analysis can be used to test second- (and higher-) order factor models in the areas of the structure of abilities, allometry, and the separation of specific and error variance estimates. In the latter area, an idea of Jöreskog's is extended to include a new conceptualization of the estimation of validity. Second-order models are placed in a hierarchy of factor analysis models, which shows how the fit of various models can be compared. The concept of discriminability is introduced to describe a situation in which two models may both be identified, and yet the goodness-of-fit for both will be the same. This problem can usually be avoided by careful design of a study. Several examples are discussed.

In a typical factor analysis, a large number of observed variables are summarized by a smaller number of unobserved factors. In a confirmatory factor analysis model or an oblique exploratory factor analysis model, the correlations among the factors are generally unconstrained. In some theoretical contexts, however, it may be possible to specify a structure which would account for the relationships among the factors. One possibility for such a structure is another factor analysis model; in this case the complete model is called a second-order factor analysis model. (Other possible models for the relationships among factors leads to the more general case of structural equation models). If the second-order factors in turn have a structure, then a third-order model may be constructed, and so on.

In this paper we will (a) discuss the relationship between the confirmatory second-order factor analysis model and some other common factor analysis models; (b) discuss the identification conditions for a second-order confirmatory factor analysis model; (c) show how second-order factor analysis can be used to get estimates of reliability and validity which are better than estimates available from other techniques; (d) provide examples of the application of the techniques discussed.

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Most previous studies in second-order factor analysis have used exploratory factor analysis; only a few have used confirmatory factor analysis. Examples of exploratory studies are those by Cattell (1963) and Humphreys (1967), who utilized ability and intelligence tests, and Gorsuch (1966), who examined a measure of anxiety. These (and other) exploratory second-order analyses proceeded in a two-step fashion, by factor analyzing the correlations among first order factors obtained using an oblique rotation.

Jöreskog (1971) conducted a confirmatory second-order factor analytic study of cognitive tests in which he found two second-order factors using ten observed variables. This model fit the data marginally well. Weeks (1980; see also Bentler & Weeks, 1980) also conducted a confirmatory second-order factor analytic study. This model of intellectual abilities consisted of one second-order factor for eight tests.

Bagozzi included a second-order factor in a structural equation model testing theories of relations among attitudes and behaviors (Bagozzi, 1981a), and in a test of an expectancy-value model of attitude (Bagozzi, 1981b).

Notation, Definitions, and Model Specification

Second-order factor models can be represented in a number of ways, and the method chosen generally will depend more on the computer program used for the analysis than anything else. We used the LISREL IV (Jöreskog & Sörbom, 1978) computer program and shall use the corresponding notational system; the RAM (McArdle & McDonald, 1984) and EQS (Bentler, 1983) systems are two others with which we are familiar in which second-order models can be easily represented.

In the second-order models, the vector of second-order factors will be represented by ξ, the first-order factors by η, and the observed variables by y. The matrix Λ_y contains the loadings of the observed variables on the first order factors, and Γ contains the loadings of the first-order on the second-order factors. The covariance matrix of the second order factors will be represented by Φ. The vector of residual variables in the first-order factors will be represented by ζ, and the unique variables in the observed variables by ϵ; the variance-covariance matrix of the residuals and uniquenesses will be called Ψ and Θ_ϵ respectively. The equation for the observed variables in terms of the first-order factors is therefore

[1] $$y = \Lambda_y \eta + \epsilon$$

and the equation for the first-order factors in terms of the second-order factors is

[2] $$\eta = \Gamma\xi + \zeta$$

The covariance matrix of the first-order factors is $\Gamma\Phi\Gamma' + \Psi$, and the observed variance-covariance matrix of the ys is

[3] $$\Sigma = \Lambda_y\,(\Gamma\Phi\Gamma' + \Psi)\Lambda'_y + \Theta\epsilon$$

The Identification Problem

A parameter of a model is said to be identified if one can solve for the value of the parameter in terms of the elements of Σ. A model is identified if each of its parameters is identified.

As with other confirmatory factor analysis models, and, more generally, with linear structural models with latent variables, it is difficult to give practical necessary and sufficient conditions for identification. Here we present some useful ways of thinking about the problem.

The identification problem for second-order factor models can be conceptualized in terms of the separate parts of the structure of the models: the first structure is that which relates the first-order factors to the second-order factors; the second structure is that which relates the observed variables to the first-order factors. For each of these parts, the usual rules of identification can be used; if each part is identified, then the model is identified.

For the first part of the model, the obvious first rule would be that if there is only one second-order factor, then there must be at least three first-order factors if the model is to be identified. If there are only three first-order factors, then this part of the model is just identified, and therefore the overall test of goodness of fit of the model does not test the second-order structure. For this reason, at least four first-order factors should be included in this situation.

An exception to this is when one or more first-order factors turn out to have nothing in common with the other first-order factors which supposedly measure the same second order factor. This can cause empirical underidentification (Kenny, 1979; Rindskopf, 1984), one symptom of which would be a zero or near-zero direct effect from the second-order to the first-order factor. This occurs, for example, if a second-order analysis is done of the data reported in Rock, Werts, Linn, and Jöreskog (1977), from a study by Warren, White, and Fuller (1974). In that study, the first order factors were Knowledge, Value Orientation, and Role Satisfaction of farm managers. A second-order analysis reveals a near-zero loading of Role Satisfaction on the second-order factor, indicating that it did not have anything in common with the other two measures.

If there are two or more oblique second-order factors, then only two first-order factors are necessary per second order factor for the model to be identified (if each first order factor loads on only one second-order factor). Again, more first-order factors are preferable, but even with only two first-order factors for each of two second-order factors, the second-order part of the model is overidentified. Empirical underidentification is, of course, still a potential problem. With more than one second-order factor, empirical underidentification can occur because of near zero effects of second-order factors (as discussed previously) or because of either near zero or near unity correlations among second-order factors. For uncorrelated second-order factors, there must be at least three first-order factors per second-order factor in order for the model to be identified.

The second part of the identification problem deals with the measured variables. As in any other structural equation model, there should be at least two measured variables per first-order factor; more are desirable. If there is only one measure of a factor, then the analyst must make an assumption about the reliability of that measure, or an assumption about how well that first-order factor measures the second-order factor. Frequently either type of assumption would be tenuous; but if the assumption made is not too unreasonable then the parameter estimates for the rest of the model should not be greatly affected.

Additional degrees of freedom may sometimes be gained in second-order models by making equality restrictions on factor loadings when sets of tests are thought to be tau-equivalent, and by making further equality restrictions on error variances when sets of tests are thought to be parallel. In some special cases, models with such restrictions will be identified when they would not be identified without the restrictions.

Comparison of Models

In the following section several types of models will be compared for one data set which is analyzed. One model is the second-order model of direct interest; the others are more- or less-restricted models.

One-Factor Model. This model is the usual one-factor model, in which all variables are allowed to load freely on the factor. The model is made identified by fixing the factor variance at one. An example is shown in Figure 1a.

Second-Order Factor Model. In this model, there is at least one second-order factor, and the first-order factors are linear combinations of second-order factors, plus a unique variable for each first-order factor. The observed variables are linear combinations of the first-order factors, plus a residual variable for each observed variable. An example of this model is shown in Figure 1b.

Group-Factor Model. In this model, there are several factors, each of which is associated with two or more observed variables, which are linear combinations of their respective factor and unique variables. An example is shown in Figure 1c.

Bi-Factor Model. This model consists of one general factor, plus group factors as in the previous model. Figure 1d shows an example. The bi-factor model is one variety of hierarchical model, in which some factors are more general than others.

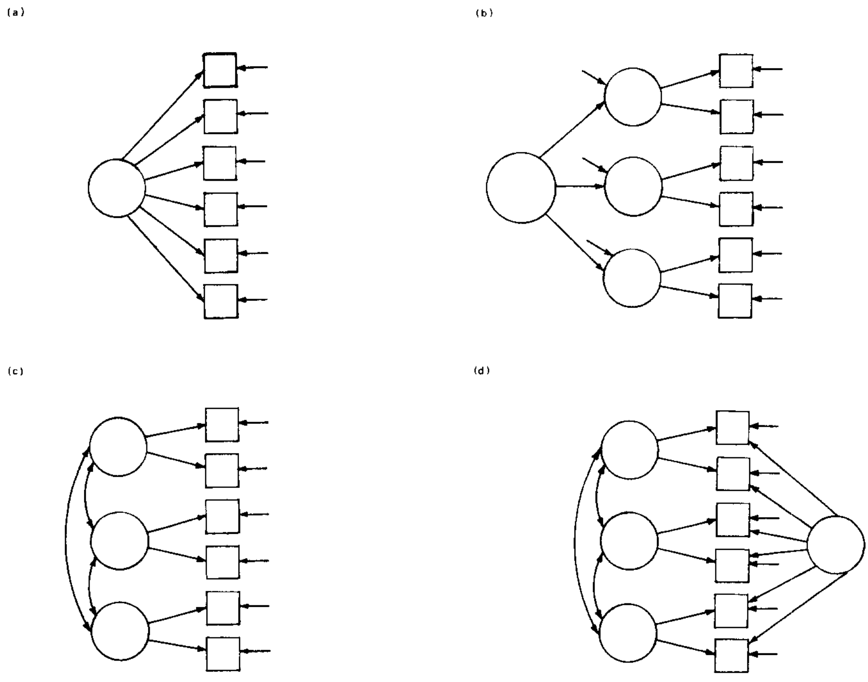

Figure 1.
A set of related factor analysis models. Circles represent factors, squares represent observed variables. For clarity, labels are not included.

The models are listed in order of decreasing restrictions: that is, the one-factor model is generally the most restricted and the bi-factor model is the least-restricted. Each model is usually a special case of the next model, in the sense that restrictions on the parameters of the one model will change it into the model above it in the series. In other words, the models are nested.

The most obvious relationship is that if there is no general factor (i.e., all loadings for the general factor are set equal to zero) in the bi-factor model, the result is a group-factor model.

To see that the second-order model is a special case of the group-factor model, it is only necessary to notice that the second-order factors put a structure on the pattern of correlations among the first-order (group) factors. This structure represents the restriction on the group factor model. This is an instance of the more general rule that a structural equation model system is a special case of a factor analysis model in which restrictions are put on the relationships among the factors. Any data which fit a structural equation model system will also fit a factor analysis model; a comparison of the goodness of fit statistics for the two models will tell whether the restrictions of the structural part of the model are reasonable.

The one-factor model is a special case of the second-order model where the unique variances of the first-order factors are set equal to zero. An examination of the resulting equations shows the equivalence of the models.

While the above demonstrates the hierarchy of the models, an additional view shows that the one-factor model can be obtained directly from the group-factor model by placing appropriate restrictions: if all correlations among the group factors are set equal to one, then the model is equivalent to a one factor model.

Obviously, any data which is consistent with one model will be consistent with a less restricted model in the hierarchy. One can choose among models which fit on the basis of theoretical plausibility, parsimony, or a statistical test of the difference.

The chain of models is only weakly ordered, in that one model might be equivalent to a following model, instead of more restricted. This will be so if and only if the two models have the same number of parameters. Then the models will be said to be not discriminable. This occurs, in general, where there are few variables per factor, and few first-order factors per second-order factor. For example, a model with one second-order factor and three first-order factors is equivalent to a group factor model. Similarly, a first-order model with correlated errors (not described above) is equivalent to a second-order model if there are only two observed measures per factor. (Gerbing & Anderson, 1984, also noted this equivalence, but incorrectly stated that it was true regardless of the number of observed measures per factor.) The choice of the form of the model in this case must be made in part on substantive grounds. One reason for preferring the second-order model is that when more measures are obtained, the second-order model will be more parsimonious. Discriminability should be considered in the design of a study, since often a small change in the design would allow the goodness-of-fit of models to be compared instead of just tested.

Comparison of Models: An Example From Allometry

Allometry deals with the size and structure of organisms. One basic aspect of allometry is the question of whether the measurement of physical organs can be summarized in one dimension ("size"), or whether more dimensions are necessary. A typical example is described in Morrison (1976, p. 288), from a study by Wright (1954). Measurements were made on 276 fowls of skull length, skull breadth, humerus length, ulna length, femur length, and tibia length. Several factor models, some of them second-order models, might be plausible for such data. The most common approach is to try to fit one-factor models to data, hoping that a size factor will be sufficient to account for variability in the measurement of different physical aspects. The next simplest model might hypothesize that there are two dimensions. If all pairs of measures, for example, were of the length and circumference (or width) or various body parts, these dimensions might be size and bulk. Another variation from the usual one factor model might allow correlated errors in the one factor model between variables which are taken from measurements of the same organ or body part.

A second-order model for these data might hypothesize a general (second-order) size factor, with specific size factors for each section or part of the body. The measurements made on the same body part would then correlate more highly than measurements made on different body parts, as would be the case with correlated errors. However, the second-order model is more restrictive, and more theoretically pleasing: it provides a reason for the higher correlations, while the correlated errors in the one factor model have no structure.

For the data set described above, a number of models might be reasonable. Unfortunately, with only two measures from three body parts several models are not discriminable: the second-order model, the one factor model with correlated errors, and the group factor model. (The chi square for testing each is 11.88 with six degrees of freedom, p = .0648).

Comparison Among Models: An Example From Intelligence Testing

This section contains a demonstration of comparisons among models, based on the data from Cattell's (1963) factorial study of intelligence. Cattell's study was undertaken to investigate his theory of crystallized and fluid intelligence factors.

The data consisted of thirteen measures of the nine ability variables from Cattell (1963) on 277 seventh- and eighth-grade children. There were two measures of each of four Thurstone Primary Abilities (Verbal, Number, Reasoning, and Spatial). The fifth Thurstone Primary Ability, Fluency, had only one measure, as did the four subtests from the Institute of Personality and Ability Testing (IPAT) Culture Fair Intelligence Test (Perceptual Series, Perceptual Classification, Matrices, and Topology). The fluency test, which would have been a single measure of a factor, was dropped from the analysis. The correlations among the remaining twelve variables are presented in Table 1.

Cattell hypothesized that a crystallized ability factor would be comprised of the Reasoning, Number, and Verbal Primary Abilities. He further posited that all IPAT subtests and perhaps the Spatial and

Table 1

<u>Correlation Matrix: Cattell Data</u>

Verbal 1	1.00											
Verbal 2	.86	1.00										
Space 1	.30	.30	1.00									
Space 2	.32	.27	.79	1.00								
Reason 1	.41	.42	.21	.23	1.00							
Reason 2	.42	.41	.25	.25	.77	1.00						
Number 1	.34	.37	.23	.16	.40	.37	1.00					
Number 2	.32	.33	.19	.14	.43	.39	.78	1.00				
IPAT Ser	.29	.30	.27	.27	.28	.24	.23	.14	1.00			
IPAT Clas	.21	.23	.29	.31	.26	.24	.18	.14	.33	1.00		
IPAT Matr	.33	.33	.28	.24	.35	.37	.32	.20	.45	.44	1.00	
IPAT Top	.25	.24	.29	.25	.23	.22	.17	.15	.29	.28	.33	1.00

Verbal Primary Abilities would constitute a fluid ability factor.

Beginning with the least-restricted model, we fit five correlated group factors, and a general factor which was uncorrelated with the group factors. The group factors consisted of one factor for each of the four pairs of Thurstone Primary Abilities, and one for the IPAT subtests. Although this model fit very well statistically (χ^2 = 23.1103, df = 32, p = .8748) it was empirically underidentified, which resulted in one negative error variance estimate.

The empirical underidentification was remedied by constraining to zero the error variance that had been negative. This produced a model that fit the data very well (χ^2 = 24.2919, df = 33, p = .8643). The parameter estimates for this model are presented in Table 2.

We note that in addition to the error variance fixed at zero (Spatial 2), the error variance for Number 2 was very close to zero. Since an error variance of zero would indicate that a variable was perfectly reliable, this indicates that there may be a problem with the model in spite of the good fit. There are several possible sources of such a problem. One is that there might be some content difference between the two tests of spatial ability, and between the two tests of number ability, making part of this model suspect. However, this hypothesis about a content difference is doubtful in view of the lack of such problems in the fit of the other models reported below.

Other possible problems include nonnormality of the data or nonlinearity; the raw data were unavailable to check whether there were such problems. Another possible source of the problem is that either the general factor or one or more specific factors were not necessary, and that collinearity among the factors caused the empirical underidentification. Since simpler models did fit the data well (see below), this last may be the most plausible reason for the problem.

Next the general factor was omitted, resulting in a group factor model with five correlated factors. The model fit well (χ^2 = 48.4331, df = 44, p = .2987); all the parameter estimates were reasonable and the standard errors were small. The parameter estimates for this model are in Table 3.

A comparison between the group-factor model and the first (bifactor) model is equivalent to a test that all the factor loadings for the general factor are equal to zero. The difference in their respective chi-squares and degrees of freedom (χ^2 = 24.1412, df = 11, p = .03) shows that the first model provides a better statistical fit to the data, while the second model does not have the interpretational problems noted for the first model.

Table 2

General Plus Group Factor Model Parameter Estimates for the Cattell Data

Variable	g	f1	f2	f3	f4	f5	Unique Variances
	Factor Loadings						
Verbal 1	.396	.884					.063
Verbal 2	.468	.764					.198
Spatial 1	.264		.767				.342
Spatial 2	.104		.995				0*
Reason 1	.354			.829			.187
Reason 2	.358			.776			.270
Number 1	.527				.645		.305
Number 2	.340				.930		.019
IPAT Ser	.419					.430	.639
IPAT Clas	.289					.543	.621
IPAT Matr	.577					.461	.454
IPAT Topol	.297					.378	.769

Factor Intercorrelations

	g	f1	f2	f3	f4	f5
g	1.					
f1	0*	1.				
f2	0*	.312	1.			
f3	0*	.387	.250	1.		
f4	0*	.228	.115	.390	1.	
f5	0*	.263	.487	.368	.047	1.

* Fixed at zero

Next a second-order model with two second-order factors was fit (see Figure 2). This puts a structure on the correlations between the five (first-order) factors. Three of the first-order factors (Verbal, Reasoning, and Number) load one of the second-order factors, while the other second-order factor is measured by Spatial and the IPAT subtests. This model fits the data well (χ^2 = 52.8030, df = 48, p = .2938). The parameter estimates for this model are in Table 4. The

Table 3

Parameter Estimates for the Group Factor Model

Variable	Factor Loadings f1	f2	f3	f4	f5	Unique Variances
Verbal 1	.923					.147
Verbal 2	.931					.133
Spatial 1		.914				.164
Spatial 2		.864				.254
Reason 1			.894			.202
Reason 2			.862			.257
Number 1				.896		.198
Number 2				.871		.242
IPAT Serial					.603	.636
IPAT Class					.581	.662
IPAT Matrices					.725	.474
IPAT Topology					.487	.763

Factor Intercorrelations

	f1	f2	f3	f4	f5
f1	1.				
f2	.358	1.			
f3	.509	.294	1.		
f4	.417	.239	.512	1.	
f5	.489	.490	.528	.377	1.

correlation between the second-order factors is .776, which is quite high; one might wish to test that both of these factors are needed. Two equivalent ways are to (a) fix the correlation at 1.0, or (b) to fit a model with only one second-order factor. Doing either of these results in a χ^2 of 61.39, with 49 *df*, $p = .11$; comparing this with the chi square from the model with two second-order factors gives a difference of 8.59 with 1 *df*. The two second-order factors are therefore both useful and are retained.

The restrictions on the structural component of the model were tested by comparing the goodness-of-fit between the second-order factor model and the group factor model. The difference ($\chi^2 = 4.3669$, $df = 4$, $p > .30$) shows that the second-order model fits no worse and

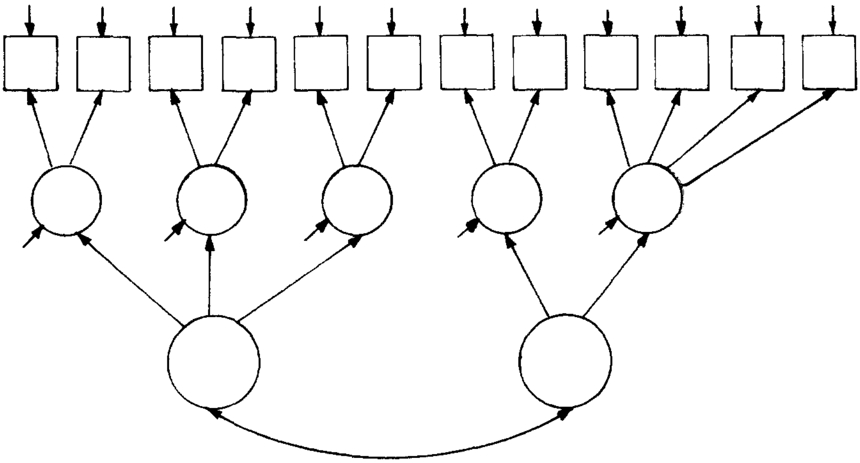

Figure 2.
Second-order factor analysis model for Cattell data (labels omitted).

would, therefore, be preferred on the grounds of parsimony.

By constraining the unique variance component for each first-order factor to zero, the second-order model becomes a two factor model, as discussed in the section above on the comparison of models. These two group factors will remain correlated, and they will be measured by the same observed variables as in the second-order model. This model did not fit the data ($\chi^2 = 632.3447$, $df = 53$).

Using Second-Order Models to Separate Reliability and Validity Estimates

One use for second-order models which has not yet been fully explored is to develop a way to estimate the reliability and validity of individual measures, and the validity of methods of measurement. Consider first the simple case of one construct (factor) being measured by three observed variables (methods). A first-order factor analysis will allow the variance of each observed variable to be separated into two parts: the common variance due to the factor and unique variance of each measurement method. The unique variance is conceptualized as containing both specific (reliable) variance and pure error, but the typical factor analysis cannot separate the two. If two or more observed measures are available for each method of measurement, however, a second-order factor model can be used to get separate estimates of the specific and error variances of each measure. This was noted by Jöreskog (1971), who did not further pursue the use of these estimates.

Table 4

Parameter Estimates for Second Order Model for Cattell Data

Variable	First Order Loadings f1	f2	f3	f4	f5	Unique Variances
Verbal 1	1.0*					.153
Verbal 2	1.016					.126
Spatial 1				1.0*		.160
Spatial 2				.940		.257
Reason 1		1.0*				.212
Reason 2		.977				.247
Number 1			1.0*			.162
Number 2			.930			.274
IPAT Ser					1.0*	.637
IPAT Class					.963	.664
IPAT Matrices					1.207	.471
IPAT Topology					.807	.764

	Loadings of First- on Second-Order Factors		Residual Variances
Verbal	.634		.444
Reason	.678		.328
Number	.566		.518
Spatial		.509	.580
IPAT		.530	.082

Notes: Fixed zeros are omitted; other fixed values are marked with asterisks. The correlation between second-order factors is .776.

Such a model might represent attempts to measure some ability using tests published by three different publishers, where each publisher has parallel forms of the test, or where tests can be split randomly in half to get scores on what should then be parallel subtests. More generally, we can conceive of models in which the observed measures of a first-order factor are not based on split halves (or thirds, etc.), repeated measures, or parallel forms. But the further the observed measures are from this situation, the more precarious is the assumption that we are really separating pure error from specific variance.

Using the notation specified earlier for the second-order model, it is easy to show that Σ, the variance-covariance matrix of the observed variables, can be expressed as

$$[4] \qquad \Lambda_y \Gamma \Phi \Gamma' \Lambda'_y + \Lambda_y \Psi \Lambda'_y + \Theta\epsilon$$

This shows directly that the variance of each observed variable is separable into three parts (when the model is identified): the first part is the common variance, the second is the specific variance, and the third the error variance. The three necessary components are found in the diagonals of the three matrices in the decomposition above. (Except for notation, this is the same decomposition which Jöreskog, 1971, presented.)

Following Jöreskog's lead, the obvious application of the separation of the variance of each observed variable into common, specific, and error components is the estimation of reliability and validity of each variable. The reliability is usually defined as the true score variance divided by the total variance. Here the true score variance is estimated as the sum of the common and specific variances. We will define the measure validity of an observed measure to be the proportion of the observed variability due to common variance, or the common variance divided by the (expected) variance of the observed variable. The reliability and measure validity of the ith observed variable are therefore estimated as the ith element of

$$[5] \qquad \text{Diag}\,(\Lambda_y \Gamma \Phi \Gamma' \Lambda'_y + \Lambda_y \Psi \Lambda'_y)/\text{Diag}\,(\Sigma)$$

and

$$[6] \qquad \text{Diag}\,(\Lambda_y \Gamma \Phi \Gamma' \Lambda'_y)/\text{Diag}\,(\Sigma)$$

respectively, where Diag(.) denotes the diagonal elements of the indicated matrix, and where the division is element-by-element on the corresponding vectors.

For the Cattell data these results are shown in Table 5. The reliability estimates for the Thurstone ability tests are larger than those for the IPAT tests. The pattern of the measure validity estimates of the variables is less clear, but in all cases the measure validity estimates are far below the reliability estimates. Because the measures of each first-order factor are not necessarily tau-equivalent, we are in the situation described above: we are not sure that the unique variance estimates are pure error.

There is more of interest here besides the properties of the observed variables. We might define the validity of a method of measurement (or method validity) as the proportion of variation in that method (i.e., first order factor) due to variation in the second-order factor. For the ith method, this is the ith component of

$$[7] \qquad \mathrm{Diag}\,(\Gamma\Phi\Gamma')/\mathrm{Diag}\,(\Gamma\Phi\Gamma' + \Psi)$$

Table 5

Variance Decomposition, Estimated Measure Validity, and Estimated Reliability of Variables in Second-Order Model of Cattell Data

	Standardized Variance Components				
Variable	Common	Specific	Unique	Reliability	Measure Validity
Verbal 1	.402	.444	.153	.846	.402
Verbal 2	.415	.458	.126	.873	.415
Spatial 1	.259	.580	.160	.839	.259
Spatial 2	.229	.512	.257	.741	.229
Reasoning 1	.460	.328	.212	.788	.460
Reasoning 2	.439	.313	.247	.752	.439
Number 1	.320	.518	.162	.838	.320
Number 2	.277	.448	.274	.725	.277
IPAT Serial	.281	.082	.637	.363	.281
IPAT Class.	.260	.076	.664	.336	.260
IPAT Matrices	.409	.119	.471	.529	.409
IPAT Topology	.183	.053	.764	.236	.183

Note: Values in first three columns are calculated from the standardized solution, not the unstandardized solution presented in Table 3.

There are thus two kinds of validities: how well the first-order factors measure the second-order factors (method validity) and how well the observed variables measure the second-order factors (measure validity). An observed measure may have low measure validity because it is unreliable or because that method of measuring the construct of interest is a poor one (has low measure validity). The estimated method validities of the five first-order factors as measures of the second-order factor which they represent are .475, .583, .382, .309, and .774.

Summary

Second order models can be more parsimonious than the usual factor analysis models. Hierarchical relationships among models often allow direct statistical comparisons of pairs of models. These hierarchies are weakly ordered, so that in some cases the models are equivalent. Planning of the variables measured in a study can often help insure that models are discriminable. This planning can be as easy as splitting tests into thirds or fourths instead of halves for scoring.

Second-order models are useful for investigating reliability and validity. These models allow the separation of specific and unique variance estimates, which in turn allow the estimation of reliability and validity of observed variables. The validity of a method of measurement can also be estimated.

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