ENDOGENOUS CURRENT ACCOUNT BALANCES IN A WORLD CGE MODEL WITH INTERNATIONAL FINANCIAL ASSETS

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ABSTRACT

This paper presents an applied computable general equilibrium world model with financial assets and endogenous current account, and capital and financial account balances. The capital and financial account equilibrium conditions, rather than exogenous rules, constrain the current account balance. International capital flows which balance the current account are constrained by supply-and-demand equilibrium conditions on the market for international debt securities, under portfolio managers’ optimizing behavior. The asset-liability structure of the financial portfolio is endogenous, and it is possible for a country-agent to have negative net financial assets. In simulations, the interaction of portfolio choices with trade supply and demand behavior leads to endogenous sign reversals in some current account balances, and it results in a different allocation of investment among regions, compared to a model with exogenously determined current account balances. In the reference scenario, this allocation generates growth that is about the same globally, but differently distributed between regions.

Keywords

World CGE model; International investment position; Current account balances; Capital and financial account

JEL codes

C68, D58, F17, F37, G11, G15
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Introduction

By the end of 2009, the U.S. net international investment position (IIP) was at –$2 737.8 billion dollars. At the same time, Japan had a positive IIP of $ 2 889 billion, and China a positive IIP of $ 1 791 billion. These impressive figures are the cumulative consequences of recurrent capital and financial account imbalances, but most world trade CGE models ignore the capital and financial counterpart of current account deficits and surpluses.

In this paper, we discuss the extension of the Walrasian equilibrium principle to the current account balance, and to the capital and financial account balance in the PEP-w-t-fin model (Lemelin et al. 2010, 2012), a worldwide recursive dynamic CGE model with international financial assets. Our model endogenizes current account balances by making explicit the international capital flows which offset the current account. Capital flows are constrained by supply-and-demand equilibrium conditions on the market for international debt securities, as determined by portfolio managers’ optimizing behavior. Each country is a single agent, owning a portfolio of assets which constitutes its net wealth. Wealth consists of financial wealth and physical assets (ownership titles to productive capital or claims on the flow of income generated by it). Financial wealth is made up of international assets and liabilities (debt). The asset-liability structure of the financial portfolio is endogenous, and it is possible for a country-agent to have negative net financial assets. Borrowing is limited, however, by the willingness of other country-agents to lend, following their own portfolio choices, and by the competition from other borrowing countries.

The cumulative consequences of capital flows on the international investment positions (IIPs) of countries define the constraints under which portfolio choices are made. Interaction between the financial and the real economy may lead, for example, to endogenous sign reversals in current account balances, a feature

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7 Government of China, State Administration of Foreign Exchange (SAFE): http://www.safe.gov.cn/
which is absent in most CGE models. Nonetheless, experiments with the model have shown that the real economy remains the principal driving force determining the simulated evolution of the world economy.

1. Financial assets and capital mobility

The main purpose of this paper is to show how a model with financial assets and endogenous current, and capital and financial account balances can contribute to our understanding of the possible evolution of the world economy. Our approach is related to the literature on capital mobility. In a careful and insightful discussion of terminology, Islam (1999) makes the distinction between «mobility of current capital»8, «mobility of savings»9, and «mobility of capital», the latter encompassing the first two. In our view, however, it is desirable to deepen the distinctions put forth by Islam, and distinguish between the mobility of physical capital, and the mobility of financial capital, where the latter implies the mobility of savings (and therefore of investment), but broadens the concept to the mobility of stocks of financial assets, including portfolio equity and foreign direct investment. The mobility of financial capital does not imply the mobility of installed physical capital, since ownership can change hands while physical capital remains immobile. In the PEP-w-t-fin model, capital mobility takes the form of financial capital mobility.

The same type of capital mobility is found in the intertemporal dynamic CGE model of Goulder and Eichengreen (1989)10. It is also found in the version of the DART recursive dynamic model applied by Hübler (2011), whose specification of foreign direct investment follows Springer (2003). In that version of DART, financial capital is allocated according to a portfolio model adapted from Goulder and Eichengreen (1989) to the recursive dynamic framework. Portfolio composition enters the household utility function. As return rates diverge from their initial values, the household can increase income (and therefore consumption), by allocating wealth differently from the initial portfolio composition. But changes in allocation are tempered because any departure from the preferred initial portfolio composition entails a utility cost, due to imperfect substitutability between domestic and foreign capital and a home bias in the household’s portfolio.

The G-Cubed model also has internationally mobile financial assets. McKibbin and Stoeckel (2009) present the G-Cubed model as a dynamic stochastic general equilibrium model (DSGE), but the stochastic element is absent (at least from the publicly available documentation to which the authors refer: McKibbin and Wilcoxen, 1999). So it is more accurate to say that it is a dynamic intertemporal

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8 «Mobility of current capital» denotes mobility of capital that is already in place and is participating or can participate in production during the current period. » (Islam, 1999, p. 3).
9 «Mobility of savings» denotes mobility of resources that become capital in the next period through purchase and installation of investment goods during this period. » (Islam, 1999, p. 3).
10 «Neither labor nor physical (as distinct from financial) capital is mobile internationally. » (p. 8).
general equilibrium model. In G-Cubed, installed physical capital is industry– and region–specific, but financial capital is perfectly mobile between industries and regions. Its allocation is driven by forward-looking investors who respond to arbitrage opportunities. Intertemporal optimization by households and firms determines saving and investment (the creation of new physical capital). Households maximize an intertemporal utility function subject to a lifetime budget constraint, and so determine the level of saving, while firms choose investment to maximize the stock market value of their equity. Current account balances are endogenous, financed by flows of assets between countries. An intertemporal budget constraint is imposed on each region: all trade deficits must eventually be repaid by future trade surpluses. With financial capital markets perfectly integrated worldwide, expected returns on assets are equalized in each period according to a set of interest arbitrage relations based on the risk-adjusted interest rate parity assumption. Household wealth portfolios include four types of assets: money, real government bonds, net claims against foreign residents, and capital. Money demand is transactions-based, proportional to the nominal value of output, where the proportion is a constant-elasticity function of the interest rate. All other assets are perfect substitutes, so that risk-adjusted return rates are equalized, taking into account the costs of adjusting capital stocks. All risk premiums are exogenous. In addition, G-Cubed includes « New Keynesian » features, such as liquidity-constrained agents and slow nominal wage adjustment. The model is designed to converge in the long run to a Ramsey neoclassical growth path.

Another model that includes the mobility of financial capital is GTAP-Dyn, the dynamic version of the GTAP model (Ianchovichina and McDougall, 2001; Ianchovichina et al., 2000). GTAP-Dyn differs from most recursive dynamic models in that it is formulated in continuous time, which is made possible by the differential equations approach of GEMPACK (General Equilibrium Modelling Package)11. GTAP-Dyn distinguishes between asset location and ownership and, hence, between physical capital and claims on physical capital; the latter are represented in the model by a single asset, equity12. In GTAP-Dyn, capital is perfectly mobile between industries within regions, but installed capital is not mobile between regions. Savings, however, are internationally mobile, through the mobility of equity assets (financial, as opposed to physical capital). Current account balances are endogenous, offset by capital account balances reflecting international flows of assets. Household wealth consists of equity. A fraction of wealth is equity in domestic capital; the rest is in the form of shares in a global trust fund which owns the fraction of capital in each region which is under foreign ownership. GTAP-Dyn does not make use of portfolio allocation theory. Portfolio shares and ownership shares are determined by a minimum cross-entropy rule

11 https://www.gtap.agecon.purdue.edu/models/current.asp. The Centre of Policy Studies (CoPS) at Monash University in Melbourne, Australia, develops and supports GEMPACK.
12 The version of MIRAGE developed by Lemelin (2009) also distinguishes between capital location and ownership, but the menu of assets includes international debt securities and the portfolio allocation mechanism is based on a micro-theoretic optimization model.
relative to initial distributions, subject to the international distribution of the stock of capital, given investment, valued at replacement cost. The distribution of capital between regions changes in time through the distribution of investment, according to the accumulation rule. Investment is entirely equity-financed, and its regional distribution is determined according to the investment theory laid out in Ianchovichina and McDougall (2001). Investment is forward looking, but it is not the result of micro-founded intertemporal optimization; rather, it is driven by a mechanism that operationalizes the economic postulate that return rates will converge in the long run if capital is mobile, albeit imperfectly\textsuperscript{13}. The model is designed to converge towards a balanced growth path, with net rates of return equal across regions, after correction for exogenous risk premiums.

To summarize, starting from the distinction laid out by Islam (1999), we focus on models where installed physical capital is immobile (at least between regions), but financial capital is internationally mobile. These include Springer (2003) and Hübler (2011) – both of which refer to Goulder and Eichengreen (1989); the G-Cubed model (McKibbin and Stoeckel, 2009); and GTAP-Dyn, the dynamic version of the GTAP model (Ianchovichina and McDougall, 2001; Ianchovichina et al., 2000). What distinguishes the model presented here is its micro-founded portfolio allocation mechanism with imperfect substitutability between assets, and the fact that portfolio choices regulate the international capital flows which offset the current account, thus making the latter endogenous. This is in contrast to the GTAP-dyn model, where portfolio shares and ownership shares are determined by a minimum cross-entropy rule relative to initial distributions. Also, the imperfect substitutability in our portfolio allocation model diverges from G-Cubed’s equalization of expected returns based on the risk-adjusted interest rate parity assumption with exogenous risk premiums. Portfolio allocation in our model is also different from the Springer version of the DART model, where portfolio composition is integrated into the household utility function, and where portfolio allocation is constrained by exogenous current account balances (to maintain comparability with

\textsuperscript{13} The model postulates that regional rates of return gradually converge to their target values, which are equal across regions except for risk premia and differences in depreciation rates. A convergence rule determines intermediate, or short-term, target rates (called expected rates in the model). The role of the intermediate target is to determine investment, i.e. the growth rate of capital, in such a way that actual rates will evolve towards the intermediate targets and, eventually, towards the long-run target rates. The rate of capital growth required to achieve convergence is computed from an aggregate CES region-wide production function. Since the aggregate production function is a simplified representation of the regional production system, the required rate of capital growth cannot be computed exactly, and the actual rate of return does not instantly converge to the intermediate target. Indeed, actual regional return rates are derived from the rental rate of capital and the price of the capital good, both determined by supply and demand equilibrium. Consequently, supply and demand equilibrium will not, in general, make the actual return rate equal to the intermediate target (expected rate). The discrepancy between the intermediate target (expected) rate of return and the actual rate is used to revise the parameters of the aggregate model of the regional economy. All adjustments are partial adjustments rather than instantaneous adjustments. The adjustment parameters are calibrated to achieve a desired speed of convergence of regional return rates to the long-run equilibrium target rates (see Golub and McDougall, 2009, p. 12-13). Overall, the GTAP-Dyn investment theory, more than an investment theory, is a model where investment is imperfectly mobile globally, and is allocated among regions in such a way that regional rates of return converge gradually, but without sudden mutations in the pattern of regional investment.
the basic version of DART). Finally, our model, like DART and GTAP-Dyn, is recursive dynamic, while G-Cubed and the Goulder-Eichengreen models are intertemporal.

The rest of the paper is organized as follows. Section 2 presents an outline of the model, then describes in more detail how endogenous asset-liability structures are determined, and examines the links of the financial to the real economy model. Next, the macroeconomic closure and reference scenario of the model are presented. Section 3 compares two variants of the model with financial assets, and a « regular » model; (in the rest of this article, we use the expression « regular model » to refer to a model with exogenous current account balances). Key aspects are investment, real GDP growth, and the evolution of net IIP, and how these relate to the endogenous current account, mainly through the trade balance. Brief closing remarks conclude the paper.

2. Model

2.1 Outline

Our model is the last-born of a family of standard models developed by our team for the Partnership for Economic Policy (PEP) Research Network. It is calibrated using the GTAP 7 data base, and IMF data on current and capital and financial accounts, and on country international investment positions (IIP).14

In its current version, the model has four industries, each producing one commodity: the primary sector, industry, services, and public administration. There are two kinds of labor, skilled and unskilled, and three other production factors: capital, land, and natural resources. The countries of the world are aggregated into 14 regions:

- Africa South of the Sahara (AfriSS)
- China (incl. Hong Kong) (ChinaHK)
- European Union Fifteen (EU15)15
- Rest of the EU (before 2007) (EUplus)
- India
- Japan
- Middle-East and North Africa (MENA)
- Latin American developing countries (LAmDev)
- Asia-Pacific developing countries (AsPaDev)
- Rest of Latin America (RoLAm)
- Rest of Asia (RoAsia)
- Rest of the world (RoW)
- Transition economies (Transit)
- United States of America (USA)

14 A complete exposition of model specification and calibration procedures is given in Lemelin et al. (2012), available online. Appendix 2 below describes the construction of bilateral debt and other financial data.
15 Member countries in 1995.
The model is admittedly quite aggregated for the time being. The moderate size of result files allows a detailed examination and facilitates diagnostics during model development. The GAMS code would nonetheless allow to apply a finer classification, both in terms of industries/commodities and in terms of regions. However, the current level of aggregation is not so coarse as to render results uninteresting.

Most CGE trade models fix current account balances exogenously, in accordance with the widely accepted view that trade policy may influence trade flows, but that current accounts are constrained by symmetric capital and financial account balances, on which trade policy has little effect (WTO, 2007). Our model was developed to make explicit the international capital flows which must take place to balance the current account implications of the simulated trade flows, and to compute the cumulative consequences of such capital flows on the international investment positions (IIP) of countries. In our model, current account balances and their capital and financial account counterparts are endogenous.

Each country or group of countries is modeled as a single agent. Every agent owns a portfolio of assets which constitutes its net wealth. There are two types of wealth: financial wealth, and physical assets. The latter are ownership titles to productive capital or, equivalently, claims on the flow of income generated by the capital. The financial component of the portfolio is made up of assets and liabilities (debt). The asset-liability structure of the financial portfolio is endogenous, and it is possible for a country-agent to have negative net financial assets (liabilities in excess of assets). The possibility of borrowing is limited, however, by the willingness of other country-agents to lend, which reflects their own portfolio choices, and by the competition from other borrowing countries. The allocation of capital among industries is determined by an investment supply and demand equilibrating mechanism which is briefly described in the latter part of subsection 2.3 below.

Country-agent wealth allocation behavior is represented in a three-tier portfolio management model (Lemelin, 2008, 2009), as illustrated in Figure 1 below. At every stage of the portfolio model, the manager maximizes a CES aggregate of the capitalized values of different assets, following an approach derived from the Decaluwé-Souissi portfolio model (Decaluwé et al., 1993; Souissi, 1994; Souissi and Decaluwé, 1997; Lemelin, 2008, 2009). The CES target function reflects the assumption that the capitalized values of different assets are not perfect substitutes in the eyes of the portfolio manager.

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16 This being a recursive dynamic model, agents are not subject to any intertemporal budget constraint.

17 Goulder and Eichengreen (1989) have two versions of their intertemporal dynamic model. In the first version, portfolio composition between domestic and foreign financial assets enters the households’ utility function as a CES function of portfolio shares. The amounts of assets, the authors argue, serve as intertemporal vehicles for savings, and consequently, they are already accounted for in the consumption program (see their endnote 17). The Goulder-Eichengreen index of portfolio satisfaction reaches its maximum value when portfolio composition is identical to the reference portfolio composition. In the second version of the model, households’ portfolio choices are independent from consumption choices, but « The independence of consumption and portfolio choices in this specification is achieved at some cost: households’ portfolio
This can be motivated by risk aversion and a desire to diversify the portfolio. It must be recognized, however, that this is an *ad hoc* representation of risk-averse behavior\(^\text{18}\). There is one feature of the model, however, which must be understood for the rest of this paper to be comprehensible, the so-called « credit margins »; this is the object of the next subsection.

\[ d \ln[\alpha / (1-\alpha)] = \sigma \ln(r_{DD} / r_{DF}). \]

It can be shown, however, that a similar rule can be derived from constrained maximization of a CES aggregate of the shares of investment *income* from domestic and foreign assets, as in Rosensweig and Taylor (1990).

\(^{18}\) A detailed presentation of the portfolio management model is found in Appendix B of Lemelin *et al.* (2012).
Figure 1. – Portfolio allocation in the PEP-w-t model

- Savings
- Wealth inherited from preceding period
- Credit margin

Amount to be allocated

Physical assets (capital ownership titles)

Capital stock and investment demand by domestic industries

Financial wealth: net financial assets + credit margin

Surplus of credit margin over liabilities

International financial assets

Amount of international liabilities = supply of debt securities

Allocation among country debt securities
2.2 Credit margins and asset-liability structures

Some countries have negative net international financial wealth (they are net international debtors). This can happen even if a country has positive savings: for example, if investment expenditures have been in excess of domestic savings (which is tantamount to saying the country has been running a current account deficit). But a portfolio model can hardly represent the allocation of a negative amount of net financial wealth.

Moreover, a region’s net financial position (assets minus liabilities) is obviously far more volatile than the underlying stocks of assets and liabilities, making a net position variable often unstable, and therefore difficult to model. So it would seem desirable to model assets and liabilities as distinct variables. But, once again, how can the model accommodate negative asset values (liabilities)? A geometric solution is illustrated in Figure 2 below.

**Figure 2 – Asset-liability structure**

The credit margin is a device to adapt the portfolio model to handle liabilities. Negative liability variables are converted to positive variables by a simple shift of origin: rather than choosing the positive amount of assets and the negative amount of liabilities, subject to net financial wealth, the regional portfolio manager chooses the positive amount of assets, and the – also positive – amount of unused credit margin...
(i.e. borrowing possibilities), subject to a constraint on the positive total of net financial wealth redefined to include the credit margin\(^{19}\). This is represented in Figure 3.

**Figure 3 – Unused credit margin as an asset**

Financial wealth is allocated between (1) a composite asset, and (2) the surplus of the credit margin over liabilities, i.e. remaining borrowing capacity: debt reduction increases the maximum amount of new loans that could be contracted, and further borrowing reduces it.

\[
FinW_{z,t} = Fasset_{z,t} + [CrdtMg_{z,t} - Debt_{z,t}]
\]

where

- \(CrdtMg_{z,t}\) is region \(z\)’s international credit margin in period \(t\), expressed in the international currency
- \(Debt_{z,t}\) is the value of region \(z\)’s international financial liabilities in period \(t\), expressed in the international currency
- \(Fasset_{z,t}\) is the value of region \(z\)’s international financial assets in period \(t\), expressed in the international currency
- \(FinW_{z,t}\) is the value of region \(z\)’s net financial wealth, including its credit margin, in period \(t\), expressed in the international currency

\(^{19}\) This construct is analogous to the « full income » concept in consumer theory with endogenous labor supply, where labor time is what is left after subtracting leisure from the consumer’s time budget, and the price of leisure is the opportunity cost of foregone labor income.
The rate of return on the composite asset is an aggregate of the interest rates on country debt securities, while the rate of return on debt reduction is the opportunity cost of debt, i.e. the interest rate on the region’s own debt.

The credit margin has been arbitrarily set in the first period to equal the sum of assets and liabilities (in other words, each country is allowed to increase its debt by the amount of its assets).

\[ CrdtMg_z = Fasset_z + Debr_z \]  

[02]

It is then assumed to grow slightly faster than the world sum of equity wealth. In spite of its simplicity, we believe that this formulation is not totally out of line with the reality of international financial markets: countries do have a total borrowing capacity, which usually exceeds their current level of debt. It is nonetheless recognized that, contrary to our specification, real total borrowing capacity is a fuzzy number, not an exact value. It is also acknowledged that the level of credit margins influences the values of the calibrated portfolio parameters, and consequently agents’ behavior in the model.

2.3 Links of the Financial to the Real Economy Model

The portfolio allocation model can be viewed as a micro-founded bridge linking the current account, through the financial and capital account, to savings and wealth, and ultimately to investment.

We begin with the balance of payments identity, which states that the sum of the current account and capital and financial account balances is zero. It follows that the counterpart of a current account surplus is a negative balance (a deficit) in the capital and financial account, which implies the acquisition of assets and/or a reduction in liabilities, and consequently an increase in the net international investment position of a country; symmetrically, the counterpart of a current account deficit is a positive balance in the capital and financial account, and a decrease in the net international investment position. Broadly speaking, the acquisition of assets can be assimilated to «lending», and the incurrence of liabilities to «borrowing». Indeed, in the current version of the model, there is a single type of international asset, labeled «debt securities», which subsume all other assets, including portfolio equity and direct investments.

So net new international lending/borrowing by a region in each period must be equal to its current account surplus/deficit. And since the acquisition of assets and the incurrence of liabilities are determined

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20 For an explanation, see Lemelin et al., 2012, Appendix B6)
21 In accordance with the Balance of Payments and International Investment Position Manual (International Monetary Fund, 2009), the acquisition of an asset is registered as a debit in the capital and financial account, while incurring a liability is registered as a credit.
22 The next step in model development would be to make foreign direct investment explicit.
in the portfolio allocation model, current accounts are endogenously constrained, regulated by regional agents’ willingness to lend and borrow. It is noteworthy that, contrary to what characterizes many other specifications, current account sign reversals are possible.

Of course, international lending and borrowing imply international income flows reflecting returns received on assets and paid on liabilities. To be consistent with the portfolio model, the current account balance includes income received on assets owned abroad, minus income paid on liabilities owned by foreigners:

\[
CAB_{z,t} = TBAL_{z,t} + e_{z,t} \left( RFass_{z,t} - RDebt_{z,t} \right)
\]

where

\(CAB_{z,t}\) : Current account balance of region \(z\)

\(e_{z,t}\) : Exchange rate; price of international currency in terms of region \(z\)’s local currency

\(RDebt_{z,t}\) : Income paid in period \(t\) by region \(z\) on its international financial liabilities, expressed in the international currency

\(RFass_{z,t}\) : Region \(z\)’s income in period \(t\) from international financial assets, expressed in the international currency

\(TBAL_{z,t}\) : Region \(z\)’s trade balance (exports, minus imports)

Net international financial income is included, however, neither in household nor in government income. For lack of systematic information on the balance sheets of households and governments, there is no obvious way of allocating the income\(^{24}\). So each region’s net international financial income is treated as an additional source of domestic savings\(^{25}\). This implies, of course, that an increase in net international financial income has no direct effect on consumption.

We now turn to the savings-investment equilibrium. From the financial model perspective, funds dedicated to current investment expenditures are the difference between (a) the desired value of physical assets (capital ownership) in the portfolio, given the optimal allocation of wealth, and (b) the current

\(^{23}\) Equation [03] parallels equation (20) in Goulder and Eichengreen (1989). Our approach to endogenizing current account balances is quite similar to theirs, although our model is recursive dynamic, while theirs is intertemporal. They, however, choose to make their equation (20) implicit from Walras’ Law, whereas we prefer to make one of the commodity-balance equations for each region implicit.

\(^{24}\) In an early version of the model, net international financial income was distributed in each region between households and the government according to their respective savings. But this distribution rule led to incongruities when the sum of household and government savings was not of the same sign as the region’s net international financial income.

\(^{25}\) Goulder and Eichengreen (1989) do not have this problem, since the government budget balances in each period and, consequently, the government has neither financial assets nor liabilities.
value of capital inherited from the past. The latter is determined under an arbitrage-based pricing mechanism (we shall return to this shortly). It can be shown that gross investment expenditures are equal to gross savings (which include capital consumption allowances, i.e. depreciation), plus net income from foreign assets, minus the current account balance (i.e. minus net new lending abroad)

\[ IT_{z,t} = SH_{z,t} + SG_{z,t} + DEP_{z,t} + e_{z,t} \left( RFass_{z,t} - RDebt_{z,t} \right) - CAB_{z,t} \]  

where

- \( DEP_{z,t} \) : Amount of depreciation (capital consumption allowance) in region \( z \)
- \( IT_{z,t} \) : Total investment expenditures in region \( z \)
- \( SG_{z,t} \) : Government savings in region \( z \)
- \( SH_{z,t} \) : Household savings in region \( z \)

Equation [04] is the traditional savings-investment equilibrium constraint, modified for the inclusion of net income from international financial assets.

Alternatively, in view of equation [03], equation [04] can be written as

\[ IT_{z,t} = SH_{z,t} + SG_{z,t} + DEP_{z,t} - TBAL_{z,t} \]  

which is the formulation usually found in CGE models\(^{27}\). But equation [05] is not merely an \textit{a priori} macroeconomic accounting identity; rather, it is an equilibrium condition, linking import demand and export supply, on one hand, to portfolio allocation behavior regarding investment expenditures on the other hand.

We now take a closer look at portfolio behavior regarding investment. Physical assets in the portfolio consist of ownership titles (equity, for short) to capital inherited from the preceding period and to capital newly created by investment. Investment is entirely financed by issuing new equity\(^{28}\). But there is no portfolio allocation mechanism between old and new equity, or among industries. All equity is considered perfectly substitutable in the eyes of the portfolio manager, so return rates on all forms of equity in a given region must be equal.

\(^{26}\) This is demonstrated in Appendix B of Lemelin \textit{et al.} (2012).

\(^{27}\) Matters would be slightly different if it were not assumed that net income from foreign assets is entirely dedicated to savings. In that case, equation [05] would be

\[ IT_{z,t} = SH_{z,t} + SG_{z,t} + DEP_{z,t} - TBAL_{z,t} - \left( 1 - \alpha_z \right) e_{z,t} \left( RFass_{z,t} - RDebt_{z,t} \right) \]

where \( \alpha_z \) is the fraction of net income from foreign assets that goes to savings.

\(^{28}\) This is similar to the GTAP Dyn model: « Another way to look at this is to imagine that firms and the trust fully distribute their net earnings as dividends to shareholders, and fund their net asset purchases entirely through new stock issues. » (Ianchovichina and McDougall, 2001, p. 13).
The allocation of investment is nonetheless the nexus where the financial and the real economy are linked. The underlying modeling postulates are that (1) the market value of an asset is equal to the present value of the income flow it is expected to generate, and (2) expectations are myopic, in the sense that current rates of return are expected to prevail forever. This second postulate is bound to be controversial. Its adoption is justified on the grounds that, unrealistic as it may be, the assumption of myopic expectations is viewed by the authors as less unrealistic than perfect foresight (it is tempting to add, especially in the light of the 2008-2009 world crisis). In any case, the perfect foresight hypothesis is not applicable in a recursive dynamic model. Myopic expectations also have the advantage of being mathematically tractable, although it would probably be feasible to introduce adaptive expectations.

The equilibrating price is the rate of return on equity. A rise in the rate of return on equity: (1) increases the optimal share of equity in wealth; (2) reduces the present value of expected income from capital ownership, and consequently reduces the value of the stock of equity (both old and new) which the portfolio must hold; and (3) raises the user cost of capital and thus diminishes investment demand. The converse is true of a fall in the rate of return. In a given region, the investment demand of each industry, in the Jung-Thörbecke (2001) style, is a constant elasticity function of Tobin’s $q^{29}$, and the latter is inversely proportional to the user cost of capital$^{30}$. So investment equity-financing is rationed among industry demands by the uniform rate of return that must be offered to holders of new equity. That rate of return determines the share which holders of new equity will receive in the income stream expected to be generated by total (old and new) capital. Accordingly, the value of equity inherited from the preceding period is revised, based on the residual income flow expected from capital, and this revision results in an equal rate of return on old and new equity$^{31}$.

### 2.4 Model parametrization, macroeconomic closure and reference scenario

Model parametrization comprises two aspects: calibration of the parameters that can be determined from the information contained in the underlying social accounting matrix (SAM), and assignment of values to the so-called free parameters that remain. The procedure begins with extracting and aggregating GTAP 7 data, and eliminating imbalances due to rounding errors. This determines key SAM flows in nominal terms. Next, free parameters are set (mostly CET and CES elasticity parameters). Then prices (other than exogenous prices) and volume variables are calibrated from the SAM data and the fixed free parameters. So far, this is a pretty usual procedure. But regarding dynamic parameters and variables (such as capital

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29 Note that, according to Tobin’s theory, investment should proceed to the point where $q = 1$ exactly. But such a specification makes the model unstable (Lemelin and Decaluwé, 2007, p. 56-57).

30 See Lemelin et al., 2012, chap. 8.

31 A formal presentation and an in-depth discussion are given in Appendix B3 of Lemelin et al. (2012).
stock and investment by industry), the GTAP 7 data is insufficient for calibrating without making structuring hypotheses. In our case, the structuring hypotheses are that the rate of depreciation, the rental rate of capital and the investment ratio (the volume of investment relative to the stock of capital) are all uniform across industries within a region in the base year. Finally, the portfolio model is calibrated from bilateral flow and stock matrices constructed from International Monetary Fund international financial data (see Appendix 2). A complete exposition of model specification and calibration procedures is given in Lemelin et al. (2012), available online.

The model’s macroeconomic closure is neoclassical, with full employment of labor and capital, savings-driven investments, and fixed regional GDP deflators and endogenous exchange rates. The overall numeraire is the exchange rate of the reference region (USA). The regional GDP deflators can be interpreted as regional numeraires, and the endogenous exchange rates as real exchange rates.

In the reference scenario, it is assumed that the economies of countries or groups of countries grow smoothly, based on demographic projections and the expected growth of GDP per capita. To take these trends into account, we define an exogenous index to update the values of variables and parameters that are expected to grow with population and GDP per capita. These consist mostly of exogenously determined volume variables: labor supply, real government expenditures, and public investments.

This index, which we call \( gdpix \), is specific to each region and increases at a rate which may vary from one period to the next. It is constructed assuming that demographic year-to-year projections are subject to less uncertainty than GDP projections, but we nonetheless consider the trend in GDP projections over several years to be acceptably reliable for the purposes of defining the reference scenario. So the GDP index grows at a rate that combines the year-to-year rate of population growth with the average rate of GDP per capita growth over the simulation horizon.

\[ (1 + g_{\text{pop}}^{\text{z,t}})(1 + \bar{g}_z^{\text{GDP,p,c}})^{-1}, \]

where \( g_{\text{pop}}^{\text{z,t}} \) is the rate of growth of population in region \( z \) from \( t \) to \( t+1 \), and \( \bar{g}_z^{\text{GDP,p,c}} \) is the average of year-to-year growth rates of per capita GDP over the simulation horizon. The authors thank David Laborde for having shared GDP forecasts and population data used in the MIRAGE model. The demographic growth rates are from the ILO projections of population. The annual growth of GDP in each region is taken from the World Bank projections.

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32 Of course, the rental rate of capital and the investment ratio are endogenous and change from period to period in the model solution. The rate of depreciation, however, is fixed.

33 More specifically, the growth rate from \( t \) to \( t+1 \) is \( (1 + g_{\text{pop}}^{\text{z,t}})(1 + \bar{g}_z^{\text{GDP,p,c}})^{-1} \), where \( g_{\text{pop}}^{\text{z,t}} \) is the rate of growth of population in region \( z \) from \( t \) to \( t+1 \), and \( \bar{g}_z^{\text{GDP,p,c}} \) is the average of year-to-year growth rates of per capita GDP over the simulation horizon.
3. Model performance and comparison

3.1 Stylized facts of the world economy

The CGE parametrization procedure ensures that the model will reproduce base-year data (here, GTAP 7 2004 data). But how well does the model fare with respect to subsequent years? The most demanding performance criterion for a CGE model is undoubtedly whether, given a proper reference scenario for exogenous variables, the model can be calibrated to track the recent history of the world economy. However, in view of the turbulences experienced in the last few years, this test would be inordinately challenging. For it must be kept in mind, when confronting the model to reality, that a CGE model – and this one is no exception – is not designed to generate crisis scenarios. So, while global imbalances are calibrated into the model, running it without shock will not normally simulate the unexpected moment when the proverbial straw breaks the camel’s back.34 Indeed, the most common way in which our model represents a crisis is by issuing an « Infeasible solution » verdict when it reaches a point where it can find no solution to the financial equations that is compatible with the rest of the model. Another point worth mentioning is that, to calibrate our model, we refrained from using total factor productivity as a Deus ex machina to align the reference scenario of regional GDP evolution on the World Bank or IMF projections (see section 2.4 above).

In addition, while our model endogenizes current account balances through the introduction of financial assets, it lacks several key elements for an in-depth analysis of global financial imbalances, as they are discussed, for example, in Obstfeld (2012). First, although our model allows leverage, the « financialization » of the world economy does not increase as rapidly in our model as observed in reality. For instance, if we take the ratio of international assets plus liabilities to GDP, we observe that it increases only by a little more than 30% worldwide over the 2004-2030 simulation period, and for no region does it increase by more than 90%, quite a contrast this with the explosive growth displayed in Obstfeld’s Figure 3. As for the ratio of gross financial flows (absolute change in assets plus absolute change in liabilities) over current account balances, it is greater than 1 worldwide, but it actually tends to fall over time, which is contrary to what is happening. The second key element that is missing is capital gains: in our model, international financial assets and liabilities are cumulated nominal value flows, and they have no price.35 Other missing features are differentiated assets (foreign direct investment, portfolio

34 Lemelin et al. (2010) have used the model to simulate the effects of the bursting of the U.S. housing bubble and subsequent subprime crisis, but not the crisis itself, which was represented as an exogenous shock.

35 International financial assets are valued in the international currency, the US dollar, which is also the model numeraire. It follows that regions experience minute capital gains or losses on their net assets as their exchange rate rises or falls (their currency is devalued or revalued – the exchange rate is the price of the international currency in term of the local currency), except for the reference region, the USA.
equity, debt securities, etc.) and risk premia. Finally, since our model is recursive dynamic, agents are not – and could not be – subject to intertemporal budget constraints; yet, it is from taking these into consideration that Obstfeld (2012, p. 6) asserts that « The key economic significance of the NIIP is that at any point in time, it limits the present value of a country’s future net export deficits. » To summarize, not only our model is not designed to simulate crises, but it does not pretend to be a tool for understanding global financial imbalances. More modestly, it takes better account of the interactions between trade and financial flows than regular models.

That being said, it remains to examine the model in the light of basic stylized facts on international investment and imbalances. Leaving aside the yet unresolved fallout of the 2008 subprime financial crisis and of the Great Recession that followed (leading to the Eurozone sovereign debt crisis), the overwhelming long term stylized facts of the world economy are the huge U.S. current account deficit, and the accelerated growth of China and other emerging economies. Before the crisis, there was a lively debate about the sustainability of the U.S. current account deficit and its cumulative result, a negative and declining net IIP. The counterpart of the U.S. negative net IIP in 2004 (the model base year and GTAP 7 reference year) was the Japanese surplus. However, while the U.S. current account balance was expected to stabilize, but not to reverse, the Japanese current account surplus was expected to decrease; the gap was to be filled by the growing surplus of emerging Asia, particularly China. As shown in the simulation results discussed below, our model reproduces these broad features.

Finally, the evolution of regional GDPs and of GDP per capital in all models largely reflects GDP forecasts and population data used to construct the gdpx parameter (see 2.4 above). Regarding real GDP per capita, our model seems to be in line with common expectations. According to the model, per capita GDP, worldwide, will roughly double between 2004 and 2030. It can be seen from Table 1 below that the ranking of regions is relatively stable, except for the rise of China, who gains three ranks thanks to a quadrupling of her real GDP per capita, and perhaps the lackluster performance of the Rest of Latin America (RoLAm)36. As anticipated, rich countries, while remaining rich, gain less GDP per capita than the rest of the world.

36 Includes Argentina, Brazil, Chile, Mexico and Uruguay.
Table 1 – Index of real GDP per capita, 2004 and 2030, according to the regular model and the model with financial assets

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>2004 Regular</th>
<th>2004 Financial assets</th>
<th>2030 Regular</th>
<th>2030 Financial assets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Index (world=100)</td>
<td>Rank</td>
<td>Index (world=100)</td>
<td>Rank</td>
<td>Index (world=100)</td>
</tr>
<tr>
<td>AfriSS</td>
<td>11</td>
<td>13</td>
<td>11</td>
<td>14</td>
<td>11</td>
</tr>
<tr>
<td>AsPaDev</td>
<td>14</td>
<td>12</td>
<td>22</td>
<td>13</td>
<td>23</td>
</tr>
<tr>
<td>ChinaHK</td>
<td>23</td>
<td>11</td>
<td>95</td>
<td>7</td>
<td>90</td>
</tr>
<tr>
<td>EU15</td>
<td>463</td>
<td>3</td>
<td>404</td>
<td>3</td>
<td>420</td>
</tr>
<tr>
<td>EUplus</td>
<td>122</td>
<td>6</td>
<td>205</td>
<td>6</td>
<td>163</td>
</tr>
<tr>
<td>India</td>
<td>9</td>
<td>14</td>
<td>23</td>
<td>12</td>
<td>20</td>
</tr>
<tr>
<td>Japan</td>
<td>587</td>
<td>2</td>
<td>484</td>
<td>2</td>
<td>510</td>
</tr>
<tr>
<td>LAmDev</td>
<td>45</td>
<td>9</td>
<td>53</td>
<td>11</td>
<td>50</td>
</tr>
<tr>
<td>MENA</td>
<td>52</td>
<td>8</td>
<td>70</td>
<td>9</td>
<td>79</td>
</tr>
<tr>
<td>RoAsia</td>
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<td>5</td>
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<td>4</td>
<td>303</td>
</tr>
<tr>
<td>RoLAm</td>
<td>68</td>
<td>7</td>
<td>69</td>
<td>10</td>
<td>73</td>
</tr>
<tr>
<td>RoW</td>
<td>269</td>
<td>4</td>
<td>236</td>
<td>5</td>
<td>242</td>
</tr>
<tr>
<td>Transit</td>
<td>39</td>
<td>10</td>
<td>85</td>
<td>8</td>
<td>91</td>
</tr>
<tr>
<td>USA</td>
<td>649</td>
<td>1</td>
<td>537</td>
<td>1</td>
<td>521</td>
</tr>
<tr>
<td>World</td>
<td>100</td>
<td></td>
<td>100</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>World (2004 $)</td>
<td>5 934</td>
<td>1</td>
<td>11 923</td>
<td>1</td>
<td>11 924</td>
</tr>
</tbody>
</table>

3.2 Comparison with the regular model

The rest of this section addresses the key issue of what more a model with financial assets can contribute to our understanding of the possible evolution of the world economy, relative to a regular CGE model. To begin to throw light on this issue, we compare two variants of our financial world trade model with a regular world trade model (the PEP-w-t model of Robichaud et al., 2012), running the reference scenario described earlier (no shock). The two variants of our financial world trade model (with financial assets) are identical, but for the portfolio elasticities: the first has relatively high values, while the second has relatively low values. In graphs and tables below, the financial model results are labeled « Hi-El » and « Lo-El »

Lemelin et al. (2011) examine the issue of portfolio elasticities in the financial world trade model. That paper uses a slightly different financial world trade model, in which regions owned shares of a world debt-security portfolio. The experiments performed in Lemelin et al. (2011) have been repeated with the current model, and the results are similar. In the simulations presented here, the elasticity of substitution between financial wealth and physical assets in regional wealth is 83 for the high elasticity variant, and 33 for the low-elasticity variant. The elasticity of substitution between the composite financial asset and the surplus of the credit margin over liabilities in the regional asset-liability structure is 60 (high) and 31 (low). The elasticity of substitution in the composite financial asset between regional debt securities is 40 (high) and 38 (low). These values may all seem high. But, to put things in proper perspective, one should keep in mind that a high elasticity of
« Lo-El », and the regular model results are labeled « Reg ». In the regular model, there are no financial assets, and the current account balance grows exogenously for each region at the same rate as labor supply and public consumption and investment, following parameter gdpix (see subsection 2.4). All three models run to the 2030 horizon following the reference scenario.

Parameter gdpix is quite different among regions. Labor supply growth rates in particular heavily influence GDP growth rates, and all the more so, given that the calibration of investment demand is based on the hypothesis that the economy is on a balanced growth path. Basic regional trends, as determined by the reference scenario, are common to the regular model and the two variants of the model with financial assets. The paths generated for each region differ from one model to the other, but these deviations are of a lesser magnitude than differences between regions.

What mainly differentiates the model with financial assets from the regular model is the imperfect substitutability between financial wealth and equity (the ownership of capital). As explained in Section 2.3, surpluses or deficits in the current account balance (CAB) translate directly into equal variations of the regional agent’s financial wealth, which, ceteris paribus, changes the ratio of equity to financial wealth in the agent’s portfolio. In the regular model, this has no consequence. But in the model with financial assets, unless the elasticity of substitution is infinite, there will be resistance to change in the composition of wealth: CAB variations are restrained or, in some cases, amplified relative to the exogenously determined CAB of the regular model, in such a way as to temper the change in the ratio of equity to financial wealth that would occur under the exogenous-CAB scenario. And, just as the adjustment of the economy to the exogenous CAB constraint is diffuse and involves all components of the economy (especially prices, including the exchange rate), so does the adjustment of the economy to the endogenous CAB financing constraint.

But the endogenous CAB financing constraint is not rigid, so part of the adjustment is absorbed through changes in the equilibrium ratio of equity to financial wealth. Since the rate of return on financial wealth is largely determined in world financial markets, the adjustment of the portfolio to a non-zero CAB is borne essentially by the exchange rate and the rate of return on equity. A rise (fall) of the rate of return on equity has two effects: (i) it increases (decreases) the optimal share of equity in the portfolio; (ii) it reduces (augments) the present value of income from capital and therefore, the market value of equity; both effects contribute towards restoring equilibrium. At the same time, a rise (fall) in the rate of return on equity raises (lowers) the user cost of capital and depresses (stimulates) investment demand, which is

38 Expressed in terms of the domestic currency.
compatible with the reduction (increase) in the total amount of savings that results from an increase (fall) in the CAB. As for the exchange rate, *ceteris paribus*, a rise (fall) increases (reduces) the proportion of financial wealth to equity in the agent’s portfolio (where financial wealth expressed in terms of the domestic currency), which is compatible with a positive (negative) CAB. That is in addition to the usual trade balance implications: a rise (fall) in the exchange rate increases (reduces) the price of imports, reduces (increases) the price of exports for foreign buyers, and favors an higher (lower) CAB.

To summarize, in the model with financial assets, the ratio of equity to financial wealth deviates less than in the regular model from its initial value, as confirmed from comparing the ratios from the models with financial assets with those that can be constructed *ex post* from the results of the regular model. We shall now examine the implications of preserving the ratio of equity to financial wealth, beginning with current account balances.

### 3.3 Current Accounts

Figures 4 and 5 present the evolution of current account balances for a sample of six of the fourteen regions in the models, as percentages of their respective GDPs. The dashed lines in the graphs represent the solution of the regular model (without financial assets).

In base year 2004, the solutions of the two variants of the financial model are different from that of the regular model, since the latter does not take into account income received from international assets and paid on international liabilities. The graphs also show that in the regular model, the current account balance as a percentage of GDP is almost constant\(^{39}\), while it changes in the models with financial assets. Indeed, four of the six cases displayed show current account balance sign reversals in the course of the simulation period. In Figure 4, which displays the trajectories of mature economies, the current account balance, initially positive, turns negative. Given the relatively sluggish growth of these economies, capital accumulation and the growth of equity wealth lag behind the accumulation of net financial wealth, which is slowed, and eventually reversed, by the fall in the initially positive CAB. Figure 5 shows different patterns: a surplus that persists (Middle East and North Africa)\(^{40}\); a slow rise from a negative to a positive current account (Africa South of the Sahara)\(^{41}\); a rise from a negative base value that tapers off before the current account becomes positive (India)\(^{42}\). In the first case, equity growth is accelerated relative to the

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39 Although the dashed lines appear to be straight lines, there are slight variations in the underlying percentages which are not perceptible in the graphs.
40 The Rest-of-Asia region (Korea, Malaysia, Taiwan and Singapore) and Transition economies follow a similar pattern, while the Rest-of-Latin-America (Argentina, Brazil, Chili, Mexico and Uruguay) progressively increases its initial surplus.
41 The Asia-Pacific developing countries follow the same pattern.
42 Latin American developing countries follow the same pattern.
regular model, thanks to a lower CAB (and less «exporting of savings»), so the ratio of equity to financial wealth falls less. For the AfriSS region, the trade balance is positive initially, but when account is taken of the net income from international assets, in the model with financial assets, the CAB is negative; in the model with financial assets, both equity and financial wealth grow slower than in the regular model, but the slow-down is more pronounced in the case of financial wealth, so the ratio of equity to financial wealth falls less. India’s negative net international investment position (IIP) becomes larger than its credit margin in the regular model, where there is no limit to borrowing; this is prevented in the model with financial assets, which requires the CAB deficit to be smaller.

Figure 4 – Current account balances as a percentage of GDP, 2004-2030
Japan, RoW region\(^43\) and EU15 region

\(^{43}\) Includes Australia and New Zealand, Canada, Switzerland, Norway, and Turkey.
3.4 Real GDP

Table 2 shows the level of real GDP\textsuperscript{44} in base year 2004 and, according to the different model solutions, in 2030. Relative to the regular model, some regions grow faster according to the models with financial assets (in particular, the Middle East and North Africa, the Rest of Asia, and transition economies\textsuperscript{45}), and others grow markedly slower (Euplus, India). Moreover, real world GDP is marginally higher at the end of the simulation period according to the model with financial assets.

\textsuperscript{44} The real GDP of each region is computed from a Fisher quantity index of aggregate value added at basic prices Real worldwide GDP is the sum of regional real GDPs. Note that the sum of regional GDPs is slightly different from world real GDP as computed from a worldwide Fisher quantity index: it is a property of the Fisher index not to be additive.

\textsuperscript{45} Mostly countries formerly part of the Soviet Union, amongst which Russia.
Table 2 – Real GDP, 2004 and 2030 (10 G $), according to the regular model and the model with financial assets

<table>
<thead>
<tr>
<th>Region</th>
<th>2004</th>
<th>2030</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Regular</td>
<td>Financial assets</td>
</tr>
<tr>
<td></td>
<td>Hi-El</td>
<td>Lo-El</td>
</tr>
<tr>
<td>AfriSS</td>
<td>48.3</td>
<td>171.7</td>
</tr>
<tr>
<td>AsPaDev</td>
<td>74.4</td>
<td>345.8</td>
</tr>
<tr>
<td>ChinaHK</td>
<td>179.6</td>
<td>1 688.3</td>
</tr>
<tr>
<td>EU15</td>
<td>1 052.8</td>
<td>1 906.3</td>
</tr>
<tr>
<td>EUplus</td>
<td>59.8</td>
<td>187.9</td>
</tr>
<tr>
<td>India</td>
<td>59.0</td>
<td>400.6</td>
</tr>
<tr>
<td>Japan</td>
<td>445.9</td>
<td>713.4</td>
</tr>
<tr>
<td>LAmDev</td>
<td>55.2</td>
<td>180.8</td>
</tr>
<tr>
<td>MENA</td>
<td>106.4</td>
<td>439.4</td>
</tr>
<tr>
<td>RoAsia</td>
<td>115.1</td>
<td>375.3</td>
</tr>
<tr>
<td>RoLAm</td>
<td>139.7</td>
<td>369.5</td>
</tr>
<tr>
<td>RoW</td>
<td>250.5</td>
<td>544.5</td>
</tr>
<tr>
<td>Transit</td>
<td>66.8</td>
<td>273.1</td>
</tr>
<tr>
<td>USA</td>
<td>1 137.5</td>
<td>2 332.7</td>
</tr>
<tr>
<td>World</td>
<td>3 790.8</td>
<td>9 929.3</td>
</tr>
</tbody>
</table>

In a model with a neoclassical closure, GDP growth is necessarily associated with factor supply growth. Now, the rate of growth of the labor supply is exogenous and is the same in all three models for any given region. So the differences in GDP growth must come from differences in the supply of capital. This is confirmed in Table 3, where the simulated 2030 real GDP, and the volumes of capital services and of labor services (Fisher quantity indexes) are expressed for each region in the form of an index relative to the solution of the regular model.
### Table 3 – Volume index of GDP, capital services and labor services, 2030, according to the model with financial assets (regular model = 100)

<table>
<thead>
<tr>
<th>Region</th>
<th>GDP</th>
<th>Capital services</th>
<th>Labor services</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hi-El</td>
<td>Lo-El</td>
<td>Hi-El</td>
</tr>
<tr>
<td>AfriSS</td>
<td>97</td>
<td>96</td>
<td>94</td>
</tr>
<tr>
<td>AsPaDev</td>
<td>101</td>
<td>101</td>
<td>102</td>
</tr>
<tr>
<td>ChinaHK</td>
<td>94</td>
<td>94</td>
<td>89</td>
</tr>
<tr>
<td>EU15</td>
<td>104</td>
<td>105</td>
<td>110</td>
</tr>
<tr>
<td>EUplus</td>
<td>79</td>
<td>75</td>
<td>62</td>
</tr>
<tr>
<td>India</td>
<td>88</td>
<td>86</td>
<td>79</td>
</tr>
<tr>
<td>Japan</td>
<td>105</td>
<td>106</td>
<td>114</td>
</tr>
<tr>
<td>LAmDev</td>
<td>95</td>
<td>93</td>
<td>90</td>
</tr>
<tr>
<td>MENA</td>
<td>113</td>
<td>121</td>
<td>120</td>
</tr>
<tr>
<td>RoAsia</td>
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<td>113</td>
<td>121</td>
</tr>
<tr>
<td>RoLaM</td>
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<td>108</td>
<td>109</td>
</tr>
<tr>
<td>RoW</td>
<td>103</td>
<td>103</td>
<td>107</td>
</tr>
<tr>
<td>Transit</td>
<td>107</td>
<td>110</td>
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</tr>
<tr>
<td>USA</td>
<td>97</td>
<td>96</td>
<td>90</td>
</tr>
<tr>
<td>World</td>
<td>100.0</td>
<td>100.3</td>
<td>100.3</td>
</tr>
</tbody>
</table>

### 3.5 GDP, Capital services and investment

Since the differences in GDP growth must come from differences in the supply of capital, it follows, given the capital accumulation rule, that there must be differences in both the volume and allocation of investment. Tables 4 and 5, parallel to Tables 2 and 3, show the level of real investment expenditures in each region in 2030, the final year of the simulation period.

Of course, the simulated 2030 levels of GDP are the result of investment, not only in a single year, but in all intervening years since base year 2004. But the 2030 pattern of investment is quite representative of the whole simulation period\(^{46}\). Indeed, it can be seen in Table 5 that the same regions that grow faster, or slower, from a larger or smaller volume of capital services in Table 3 also have more, or less, investment. But this observation merely displaces the question of why GDP growth is different in the simulations with financial assets: why are investment patterns different?

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\(^{46}\) The single exception is the AsPaDev region: its real investment expenditures are lower in the model with financial assets until about 2020, after which they become higher.
Table 4 – Real investment expenditures, 2004 and 2030 (10 G $), according to the regular model and the model with financial assets

<table>
<thead>
<tr>
<th></th>
<th>2004</th>
<th>2030</th>
<th>Elastici</th>
<th>Hi-El</th>
<th>Lo-El</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Regular</td>
<td>Financial assets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AfriSS</td>
<td>23.6</td>
<td>88.5</td>
<td>83.1</td>
<td>83.1</td>
<td></td>
</tr>
<tr>
<td>AsPaDev</td>
<td>39.2</td>
<td>172.6</td>
<td>183.1</td>
<td>181.3</td>
<td></td>
</tr>
<tr>
<td>ChinaHK</td>
<td>55.0</td>
<td>503.9</td>
<td>435.5</td>
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</tr>
<tr>
<td>EU15</td>
<td>191.4</td>
<td>379.6</td>
<td>477.3</td>
<td>485.3</td>
<td></td>
</tr>
<tr>
<td>EUplus</td>
<td>26.5</td>
<td>80.2</td>
<td>27.7</td>
<td>24.7</td>
<td></td>
</tr>
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<td>110.2</td>
<td>81.5</td>
<td>77.3</td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>79.4</td>
<td>131.4</td>
<td>175.7</td>
<td>175.7</td>
<td></td>
</tr>
<tr>
<td>LAmDev</td>
<td>17.0</td>
<td>58.7</td>
<td>51.7</td>
<td>49.6</td>
<td></td>
</tr>
<tr>
<td>MENA</td>
<td>33.4</td>
<td>135.0</td>
<td>172.2</td>
<td>196.2</td>
<td></td>
</tr>
<tr>
<td>RoAsia</td>
<td>27.1</td>
<td>89.1</td>
<td>121.4</td>
<td>130.6</td>
<td></td>
</tr>
<tr>
<td>RoLAm</td>
<td>37.1</td>
<td>112.6</td>
<td>128.5</td>
<td>140.2</td>
<td></td>
</tr>
<tr>
<td>RoW</td>
<td>46.3</td>
<td>105.1</td>
<td>122.2</td>
<td>122.5</td>
<td></td>
</tr>
<tr>
<td>Transit</td>
<td>10.1</td>
<td>39.8</td>
<td>50.1</td>
<td>52.4</td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>133.3</td>
<td>357.0</td>
<td>283.8</td>
<td>266.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 5 – Real investment expenditure index, 2030, according to the model with financial assets (Regular model = 100)

<table>
<thead>
<tr>
<th></th>
<th>2030</th>
<th>Elastici</th>
<th>Hi-El</th>
<th>Lo-El</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Regular</td>
<td>Financial assets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AfriSS</td>
<td>100</td>
<td>94</td>
<td>94</td>
<td></td>
</tr>
<tr>
<td>AsPaDev</td>
<td>100</td>
<td>106</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td>ChinaHK</td>
<td>100</td>
<td>86</td>
<td>87</td>
<td></td>
</tr>
<tr>
<td>EU15</td>
<td>100</td>
<td>126</td>
<td>128</td>
<td></td>
</tr>
<tr>
<td>EUplus</td>
<td>100</td>
<td>35</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>100</td>
<td>74</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>100</td>
<td>134</td>
<td>134</td>
<td></td>
</tr>
<tr>
<td>LAmDev</td>
<td>100</td>
<td>88</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>MENA</td>
<td>100</td>
<td>128</td>
<td>145</td>
<td></td>
</tr>
<tr>
<td>RoAsia</td>
<td>100</td>
<td>136</td>
<td>147</td>
<td></td>
</tr>
<tr>
<td>RoLAm</td>
<td>100</td>
<td>114</td>
<td>124</td>
<td></td>
</tr>
<tr>
<td>RoW</td>
<td>100</td>
<td>116</td>
<td>117</td>
<td></td>
</tr>
<tr>
<td>Transit</td>
<td>100</td>
<td>126</td>
<td>131</td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>100</td>
<td>80</td>
<td>75</td>
<td></td>
</tr>
</tbody>
</table>

3.6 INVESTMENT AND CURRENT ACCOUNT BALANCES

We now show how this comes through the differential evolution of the current account balances, which are endogenous in the two variants of the financial world trade model, but exogenous in the regular world trade model.
It was recalled at the outset that the counterpart of the current account is the capital and financial account: regions with current account surpluses «lend», in the broadest sense, to regions with deficits. Indeed, in CGE models, foreign savings in a region are identically equal to minus the current account balance. It follows that, *ceteris paribus*, total (domestic and foreign) savings increase or fall when the current account balance falls or increases. And the same is true of investments in a model where the latter are savings-driven.

Returning to equation [03], the current account balance in the financial model consists of the trade balance, plus net income on international financial assets. But incomes received and paid on international assets and liabilities are mostly the result of past trade surpluses and deficits. Moreover, given the financial model specification, net income on international assets is not directly related to investment expenditures, as shown in equation [05]\(^47\). So we focus our attention on trade balances.

### 3.7 Trade Balances

Figure 6 displays the evolution of global trade imbalances as a percentage of world GDP. We measure global trade imbalances as the sum of trade surpluses (expressed in terms of the reference currency), which is identically equal to the sum of the absolute values of trade deficits.

**Figure 6 – Global trade imbalances as a percentage of world GDP, 2004-2030**

\(^47\) This is no issue in the standard model, since the current account balance is identical to the trade balance.
Figure 6 shows that simulations generated by our financial assets model involve less trade imbalances globally than those generated by the regular model, and that the difference is greater with weaker portfolio elasticities (Lo-EI). The models with financial assets, in reducing trade imbalances, also reallocate savings, and consequently investment. Let’s take a closer look at that reallocation.

In Table 6, we decompose the differences in simulated real investment expenditures for 2030, by region, between the regular world trade model solution and the solutions of the models with financial assets. Equation [05] can be written as

\[ \text{Investment} = \text{Domestic savings} - \text{Trade balance} \]  

[06]

To make investment expenditures comparable across periods and between simulations, equation [06] is divided throughout by the (domestic) price of the investment good, so that equation [06] is formulated in real terms.\(^{48}\) Equation [06] implies

\[ \Delta \text{Investment} = \Delta \text{Domestic savings} - \Delta \text{Trade balance} \]  

[07]

where the \(\Delta\) refers to the simulated value of a variable in the financial model, minus its simulated value in the regular one. Dividing through by the absolute value of the difference in investment yields

\[ \frac{\Delta \text{Investment}}{|\Delta \text{Investment}|} = \frac{\Delta \text{Domestic savings}}{|\Delta \text{Investment}|} - \frac{\Delta \text{Trade balance}}{|\Delta \text{Investment}|} \]  

[08]

When \(\frac{\Delta \text{Trade balance}}{|\Delta \text{Investment}|} < 0\), then the contribution of the difference in foreign savings to the difference in total savings is negative, and, with very few exceptions, this is associated with a negative difference in investment (or, to put it brashly, the difference in foreign savings dominates). In Table 6, the first column presents the simulated amount of real investment expenditures according to the regular model; the next two columns present the differences of each of the two variants of the financial world trade model with respect to the regular one; the two final columns present the ratio \(\frac{\Delta \text{Trade balance}}{|\Delta \text{Investment}|}\) in percentage form.

As expected, when the values in the last two columns are negative, the corresponding values in the second and third column are also negative, with the single exception of Africa South of the Sahara (AfriSS). And

\(^{48}\) The price of the capital good is based on an exact price index, meaning that the price index is mathematically consistent with the aggregator function of goods into increments to the capital stock. However, the metric imposed on the capital stock in the calibration procedure, while consistent within each region, is not necessarily consistent between regions, so it would not be appropriate to sum regional real investment expenditures to obtain a worldwide total. Note in particular that, after dividing trade balances by the price of investment, the resulting figures must be viewed as foreign savings in real terms for the region, rather than as real trade balances, because the world sum is no longer zero.
we can repeat the same remark for Table 6 as we made about Table 5: the final year of the simulation, 2030, is representative of the intervening years. The single exception is the AsPaDev region, where the contribution of foreign savings to investment is negative at first, but turns positive around 2015-2016.

### Table 6 – Investment and trade balances, 2030, according to the regular model and the model with financial assets

<table>
<thead>
<tr>
<th>Region</th>
<th>Regular model Investment (10 G$)</th>
<th>Δ Investment Fin. Assets – Regular (10 G$)</th>
<th>Δ Trade balance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hi-El</td>
<td>Lo-El</td>
<td>Hi-El</td>
</tr>
<tr>
<td>AfriSS</td>
<td>88.5</td>
<td>-5.4</td>
<td>-5.4</td>
</tr>
<tr>
<td>AsPaDev</td>
<td>172.6</td>
<td>10.5</td>
<td>8.7</td>
</tr>
<tr>
<td>ChinaHK</td>
<td>503.9</td>
<td>-68.3</td>
<td>-67.3</td>
</tr>
<tr>
<td>EU15</td>
<td>379.6</td>
<td>97.7</td>
<td>105.7</td>
</tr>
<tr>
<td>EUplus</td>
<td>80.2</td>
<td>-52.4</td>
<td>-55.5</td>
</tr>
<tr>
<td>India</td>
<td>110.2</td>
<td>-28.7</td>
<td>-33.0</td>
</tr>
<tr>
<td>Japan</td>
<td>131.4</td>
<td>44.3</td>
<td>44.3</td>
</tr>
<tr>
<td>LAmDev</td>
<td>58.7</td>
<td>-7.0</td>
<td>-9.1</td>
</tr>
<tr>
<td>MENA</td>
<td>135.0</td>
<td>37.3</td>
<td>61.2</td>
</tr>
<tr>
<td>RoAsia</td>
<td>89.1</td>
<td>32.3</td>
<td>41.5</td>
</tr>
<tr>
<td>RoLAm</td>
<td>112.6</td>
<td>15.9</td>
<td>27.5</td>
</tr>
<tr>
<td>RoW</td>
<td>105.1</td>
<td>17.1</td>
<td>17.5</td>
</tr>
<tr>
<td>Transit</td>
<td>39.8</td>
<td>10.3</td>
<td>12.5</td>
</tr>
<tr>
<td>USA</td>
<td>357.0</td>
<td>-73.1</td>
<td>-91.0</td>
</tr>
</tbody>
</table>

Returning to Table 3, we can summarize our analysis so far as follows. The model with financial assets takes into account agents’ behavior with respect to the capital and financial account counterpart of the current account; this has the effect of reducing the amplitude of global trade imbalances relative to those simulated in the regular model, resulting in a reallocation of investment expenditures. The cumulative effect is reflected in capital stocks and in the volume of capital services and, ultimately, in the level of real GDP.

### 3.8 Cumulative Trade Balances and Net International Investment Positions (IIP)

We now turn to the cumulative financial implications of the evolution of current account balances. In Figure 7, we consider the cumulative trade balances of China, the EU15 and the USA. The dashed lines are the simulation results of the regular model. We see that, at least for these three big players, the two models with financial assets yield similar trajectories. The US cumulative trade deficits are lower in the models with financial assets, while the Chinese cumulative surplus is larger; the EU15, which have a virtually balanced trade according to the regular model, develop a deficit according to the model with financial assets.
However, the cumulative trade balance ignores income received and paid on international assets and liabilities. Our calculations also fail to take into account the effect of surpluses and deficits incurred prior to base year 2004. So we next look at the net international investment position (IIP).

Figure 8 tracks the evolution of the net IIP of the same three big players according to the three models. Compared to Figure 7, China’s net income from international assets increases her net IIP above her cumulative trade surplus. The case of the USA is a mirror image: the reduction of the cumulative trade deficit (relative to the regular model) observed in Figure 7 is cancelled by net income paid on international liabilities.
Figure 8 – Net international investment position (IIP), China, EU15 and USA, 2004-2030 (10 G$)

Figure 9 presents the evolution of net IIP for various regions of Asia. Most notable is the difference between models regarding Japan. In the regular model, Japan’s positive IIP follows a straight-line indefinite growth path, whereas in the models with financial assets, it reaches a peak in 2026, and then begins to decline.
Concerning the sensitivity of model results to the choice of portfolio elasticities, we have seen that, in the simulations examined here, the two variants of the model with financial assets yield results that are relatively close in most respects, and qualitatively similar. These results are also closer to one another than to the solution of the model without financial assets. However, it was shown in Lemelin *et al.* (2011), that sensitivity is somewhat greater when tests are not restricted to sets of elasticities that allow the model to run up to 2030.

**Concluding remarks**

The PEP-w-t-fin model presented here should still be considered a prototype of a world CGE model with financial assets. This model extends the Walrasian general equilibrium principle to the endogenous determination of the complementary current account and capital and financial account balances. It also examines the cumulative implications of trade flows on net international investment positions.

We compare the model with financial assets and a regular model, and conclude that the former yields results that are consistent with economic theory. The capital and financial account equilibrium conditions, rather than exogenous rules, constrain the current account balance. The interaction of portfolio choices with trade supply and demand behavior results in a different allocation of investment among regions, compared to a model with exogenously determined current account balances. Under the reference
scenario, this allocation generates growth that is about the same globally, but differently distributed between regions.

In terms of modelling practice, we have shown that endogenizing the current account balance and taking into account international financial assets is a feasible modelling approach, which can be implemented at a reasonable cost. The proposed modelling approach is applicable to any issue that would otherwise be tackled with a regular CGE world trade model. Generally, our model’s fundamental strengths and limitations are the same as those of other CGE world trade models, except for the fact that the restrictive assumption of exogenous current account balances is removed. On the other hand, removing that restriction requires the introduction of a portfolio allocation model which is a highly stylized representation of world financial markets, with elasticity parameters that have been given a priori values. Moreover, the calibration of the portfolio model is based on financial stock and flow data that is far less reliable and consistent than the real GTAP 7 database (see Appendix 2). That being said, our comparisons show that the model with financial assets, while generating economically meaningful differences, does not deviate from the regular one to the point where the credibility of its results could be called into question.

Our main policy-relevant conclusion refers to the kind of model upon which policy-makers should base their decisions, given that several CGE models are developed precisely for that purpose. To be more specific, we say that, insofar as policy-makers rely on the results of CGE world trade models, those with exogenously determined current account balances are unduly restrictive. Such restrictions are likely to be particularly damaging in applications involving changes in the pattern of world trade, such as the lowering of trade barriers, the creation of custom unions, or the implementation of environmental regulations that alter comparative advantages. There are several reasons for this. First, imposing exogenous current account balances amounts to assuming that changes in trade will not affect the evolution of trade balances, an assumption that is unfounded. Second, exogenous current account balances fix the (positive or negative) contribution of foreign savings to investment in each region, so that only variations in the amount of local savings can change the worldwide pattern of investment and, consequently, of growth (see section 3.7). Finally, to impose exogenous current account balances is to ignore the issue of their sustainability; it takes no account of the possibility that the cumulative effect of recurring current account surpluses or deficits may lead to self-correcting changes (see section 3.3), or to deepening imbalances and financially unstable or even untenable situations (see section 3.8).
Role of the funding source

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References


APPENDIX 1: ELASTICITY OF SUBSTITUTION BETWEEN ASSETS
AND ELASTICITY OF DEMAND FOR INDIVIDUAL ASSETS

A high elasticity of substitution between assets does not imply such a high elasticity of demand for individual assets with respect to rates of return. We illustrate this property referring to the allocation of a region’s international financial wealth between debt securities issued by other regions. The portfolio manager’s problem is

Maximize \[\sum_{zj} \beta_{z,zj}^{PTF-W} \left[ (1 + RR_{zj,t}^{Debt}) PtfInv_{z,zj,t}^{PTF-W} \right] \rho_{z}^{PTF-W} \] \[= \frac{1}{\rho_{z}^{PTF-W}} \]

subject to

\[Fasset_{z,t} = \sum_{zj} PtfInv_{z,zj,t}^{PTF-W} \]

and the equilibrium condition

\[\sum_{z} PtfInv_{z,zj,t}^{PTF-W} = Debt_{zj,t}^{PTF-W} \]

where

\[Debt_{zj,t}^{PTF-W} \] is the value of region \(zj\)’s international financial liabilities in period \(t\), expressed in the international currency

\[Fasset_{z,t}^{PTF-W} \] is the value of region \(z\)’s international financial assets in period \(t\), expressed in the international currency

\[PtfInv_{z,zj,t}^{PTF-W} \] is the value of region \(z\)’s stock of international portfolio investment in region \(zj\)’s debt securities in period \(t\), expressed in the international currency

\[RR_{zj,t}^{Debt} \] is the rate of interest paid in period \(t\) on region \(zj\)’s international financial liabilities

with elasticity of substitution

\[\sigma_{z}^{PTF-W} = \frac{1}{\rho_{z}^{PTF-W} + 1}, \quad 0 < \sigma_{z}^{PTF-W} < \infty, \text{ which implies } \rho_{z}^{PTF-W} = \frac{1 - \sigma_{z}^{PTF-W}}{\sigma_{z}^{PTF-W}} \text{ and } \]

\[-1 < \rho_{z}^{PTF-W} < \infty \]

Solving the problem leads to optimal portfolio shares.
\[
\frac{w_{z,j,t}}{F_{z,t}} = \frac{PtfInv_{z,j,t}}{Fasset_{z,t}} = \frac{\left(\beta_{z,j}^{PTF_{-W}}\right)^{\sigma_{PTF_{-W}}}}{\sum_{zj} \left(\beta_{z,j}^{PTF_{-W}}\right)^{\sigma_{PTF_{-W}}} \left(1 + R_{Debt}^{zj,t}\right)^{\sigma_{PTF_{-W}}}} \left(1 + R_{Debt}^{zj,t}\right)^{\sigma_{PTF_{-W}}-1}
\]

[08]

It follows that the elasticity of share \(w_{z,j,t}\) with respect to rate of return \(R_{Debt}^{zj,t}\) is

\[
\frac{\partial w_{z,j,t}}{\partial R_{Debt}^{zj,t}} = \left(1 - w_{z,j,t}\right) \left(\frac{R_{Debt}^{zj,t}}{\left(1 + R_{Debt}^{zj,t}\right)^{\sigma_{PTF_{-W}}}}\right)
\]

[09]

It can be seen that even a high value of \(\sigma_{PTF_{-W}}\) will result in a moderate elasticity, because it is multiplied by \(\frac{R_{Debt}^{zj,t}}{\left(1 + R_{Debt}^{zj,t}\right)^{\sigma_{PTF_{-W}}}}\). For example, if an asset represents 1% of the portfolio, a substitution elasticity of 100, with a rate of return of 3%, yields a share elasticity of less than 2.9.
APPENDIX 2: CONSTRUCTION OF BILATERAL DEBT AND OTHER FINANCIAL DATA

The international financial data to calibrate the portfolio model was constructed mainly from the International Monetary Fund’s International Financial Statistics (IFS), with complementary information from the Balance of Payments Statistics (BOPS)\(^{49}\) (http://www.imf.org/external/index.htm). The data was examined to correct for minor errors, and to standardize country names. It was then aggregated and balanced, following the method described in Lemelin (2011)\(^{50}\). The following paragraphs provide additional information on the preparation of financial data.

A2.1 Flow data

Flow data retrieved include:

**Current account**
- Direct investment income received
- Direct investment income paid
- Portfolio and other investment income received
- Portfolio and other investment income paid
  - Portfolio investment income received
  - Portfolio investment income paid
  - Other investment income received
  - Other investment income paid

**Capital and financial account**

**Financial account**\(^{51}\)
- Net increase in FDI assets
- Net increase in FDI liabilities
- Net increase in portfolio, financial derivatives and other investment assets
- Net increase in portfolio, financial derivatives and other investment liabilities
  - Net increase in portfolio assets
  - Net increase in portfolio liabilities
  - Net increase in financial derivatives (assets)
  - Net increase in financial derivatives (liabilities)
  - Net increase in other investment assets
  - Net increase in other investment liabilities
- Net increase in total reserves minus gold

---

\(^{49}\) In the IFS balance of payments data, the income component of the current account comprises (1) investment income (consisting of direct investment income, portfolio investment income, and other investment income), and (2) compensation of employees (IMF data browser guide, http://www.imfstatistics.org/imf/IFSIntTr.htm). The breakdown was obtained from the BOPS.

\(^{50}\) Note that Lemelin (2011) applies the balancing method to the Lane and Milesi-Ferretti (2006) data, whereas the data used here is IMF official data.

\(^{51}\) Net flows are calculated as credits, minus debits. Recall that the sale of an asset or the issuance of a liability is a credit, while the acquisition of an asset or the redemption of a liability is a debit (or use of funds).
Note that the above list has no capital and financial account data for the capital account part (capital transfers, and acquisitions and disposal of non-produced financial assets). Capital account flows are absent from PEP-w-t-fin, although they could be included as exogenous variables.

In order to be used for calibration, the flow data must be balanced: globally, income paid of each category must be equal to income received, and asset acquisition of each category must be equal to disposition. Balancing can be done either before, or after aggregation. On that issue, Lemelin (2011) concludes: «What does matter to some extent, however, is the sequencing of adjustment and aggregation. Aggregating previously adjusted data produces results that are different from adjusting previously aggregated data. The latter yields an adjusted matrix that is significantly closer to the unadjusted aggregated data, according to all closeness criteria except MAD. The choice of a sequence depends on the purpose of the balancing exercise.» (p. 14). Tests confirmed that the same conclusion applies to flow data under consideration here, with respect to both country aggregation and flow category aggregation. Accordingly, before balancing, country data was aggregated to the 14 model regions, based on the definition of the 14 regions in terms of the GTAP geography, and on the correspondence of IMF countries to GTAP regions, and the flow categories enumerated above were aggregated to:

**Current account**
- Direct investment income received
- Direct investment income paid
- Portfolio and other investment income received
- Portfolio and other investment income paid

**Capital and financial account**

**Financial account**
- Net increase in FDI assets
- Net increase in FDI liabilities
- Net increase in portfolio, financial derivatives, other investment assets and reserve assets
- Net increase in portfolio, financial derivatives and other investment liabilities

The separation between direct investment and other forms of investment was maintained in view of making FDI explicit in PEP-w-t-fin in the near future. Flow data was balanced following the basic model described in Lemelin (2011). However, financial account flow data is not required for the calibration of the current version of PEP-w-t-fin, since capital and financial account flows are aggregated to changes in total assets and changes in total liabilities, and are calibrated from current account balances, given the endogenous asset-liability structure of the financial portfolio, and assets and liabilities of the preceding period. So only current account data were retained, after being aggregated to income paid and income received.

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52 The reasons for aggregating reserve assets with portfolio assets are given in Lemelin (2011, p. 2).
A2.2 Stock data

Stock data retrieved include the following functional categories:

- Foreign direct investment (FDI) assets
- Foreign direct investment (FDI) liabilities
- Portfolio equity assets
- Portfolio equity liabilities
- Debt assets (portfolio debt + other investment)
- Debt liabilities (portfolio debt + other investment)
- Financial derivatives (assets)
- Financial derivatives (liabilities)
- Total reserves minus gold

To be used for calibration, the stock data must be balanced: globally, for each functional category, the sum of assets must be equal to the sum of liabilities (after aggregating Total reserves minus gold with Debt assets\(^{53}\)). The stocks had previously been balanced, both for detailed country data, and for aggregated regional data, following the basic model in Lemelin (2011). What remained to be done was to construct bilateral data.

To do this, the previously balanced data\(^ {54}\) was aggregated to:

- \(\text{Debt}_{Oz}\): the value of country \(z\)’s international financial liabilities in the base year, expressed in the international currency;
- \(\text{Fasset}_{Oz}\): the value of country \(z\)’s international financial assets in the base year, expressed in the international currency.

This amounts to aggregating each country’s IIP to one asset and one liability\(^ {55}\). These two sets of values constitute the marginal totals of the bilateral debt matrix. To fill in the matrix, the following procedure was applied:

- A bi-proportional bilateral asset-debt matrix was created from the marginal totals. But a bi-proportional matrix has non-zero diagonal elements, which, in the present case, would imply that every country possesses a fraction of its own debt.
- So all diagonal elements in the bi-proportional matrix were set to zero\(^ {56}\). Then the matrix columns were rescaled so that the sum of each column be equal to \(\text{Debt}_{Oz}\), the total debt of the corresponding country (the stock data having been previously balanced, the world total of \(\text{Debt}_{Oz}\)

---

\(^{53}\) The reasons for aggregating reserve assets with portfolio assets are given in Lemelin (2011, p. 2).

\(^{54}\) It might have been more consistent with the Lemelin (2011) conclusion to aggregate the functional categories before balancing the data.

\(^{55}\) It is important to keep in mind that, in this paper, « Debt » designates IIP liabilities of whatever nature, not government debt.

\(^{56}\) In the country-level data, the People’s Republic of China is distinct from Hong Kong. To be consistent, cross-ownership of debt securities between China and Hong Kong was also set to zero.
is equal to the world total of $\text{Fasset}_z \)$. The resulting matrix column sums are consistent with column totals $\text{Debt}_z$, but row sums are no longer consistent with row totals $\text{Fasset}_z$.

- This *a priori* matrix was then adjusted to row and column totals $\text{Fasset}_z$ and $\text{Debt}_z$, following the minimum information gain principle (also known as minimum cross-entropy), which is equivalent to RAS in this case.

An attempt was made initially to apply the above procedure to data previously aggregated at the 14-region level. But the adjustment problem is then infeasible, because the total amount of EU15 assets is larger than the total amount of all other regions’ debt. Obviously, this is due to the fact that EU members own debt securities issued by other EU members, and that, consequently, the EU15 region as a whole owns part of its own debt. This is why the construction of bilateral debt data proceeded at the country level. The bilateral country debt matrix was then aggregated to the 14 regions of PEP-w-t-fin. In the adjusted matrix, the USA does not own any US debt securities, neither does India own Indian securities, or China own Chinese securities, or Japan own Japanese securities. The EU15 region, however, owns part of the EU15 debt, even though, in the underlying country data, no country possesses any part of its own debt; the same is true of other multi-country regions.

Finally, once aggregated to the regional level, the bilateral matrix was no longer exactly balanced, due to rounding errors, and increasing the precision of the adjustment seemed computationally impossible. Consequently, a final round of adjustment was applied to the regional bilateral debt matrix, once again applying the minimum information gain principle.