Measuring Poverty in a Multidimensional Perspective: A Review of Literature

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1 Introduction

A poverty measure is an index synthesizing all information available about the poor population. Given a distribution of one or several indicators of individual's welfare and a poverty line (suitably adjusted, if need be, for differences in individual needs, family composition, and prices faced), such a measure yields a single index that summarizes the extent of poverty generated by this distribution. Specifying a poverty measure is not, however, a simple task. Indeed, many conceptual and methodological issues should be addressed before, such as: What individual welfare indicators should be retained? Who is really poor and why? How can the set of information describing the poor population be synthesized into a synthetic poverty measure? The economic literature dealing with these questions emphasizes that it is often hard, if not impossible, to find a consensus on the process yielding an appropriate poverty index. This diversity of opinions can be attributed to the fact that poverty is not an objective concept. On the contrary, it is a complex notion, the normative analysis of which inevitably leads to a choice of ethical criteria. These latter, though they allow us to delimit the concept of poverty, distance us from any universal agreement on the results of the measure selected for poverty analysis.

The rest of the paper is structured as follows. Section 2 develops some of the methodologies that have been applied to various aspects of poverty without utilizing an axiomatic approach. Section 3 presents the theoretical framework of multidimensional poverty measures based on the axiomatic approach. Finally, Section 4 concludes.

2 Multidimensional Poverty Measures: a Non-axiomatic Approach

Empirical studies of poverty are usually based on one-dimensional indicators of individual welfare, such as income (or total expenditure) per capita or per equivalent adult. When more than a single dimension of welfare is considered outside of the axiomatic approach, poverty comparisons are either based on a combination of a series of indicators that have been previously aggregated across individuals (Section 2.1) or on individual data that allow the retained welfare indicators to be aggregated at the individual level first, and then across individuals (Section 2.2).

2.1 The Use of Several Aggregate Welfare Indicators

A simple way to account for the multidimensional aspect of poverty is to examine several aggregated welfare indicators simultaneously. This path was followed by Adams and Page (2001), for example. They assert that the international community is increasingly sensitive to other, non-monetary, aspects of poverty, such as education, life expectancy at birth, and health, in addition to its monetary side. Using aggregate data from the World Bank that is available for several countries in the Middle East and North Africa, these authors compare the performances recorded for each indicator in several countries in this region. They observe that there is no clear relationship between a reduction in monetary poverty and an improvement in other welfare indicators. A country may, for example, have a high rate of monetary poverty alongside a high rate of education, and vice versa. Comparison between countries is thus not possible unless all indicators are aggregated into a single synthetic index.

The Human Development Report published by the UNDP (1997) states that, while pointing to a crucial element of poverty, a lack of income only provides part of the picture in terms of the many factors that impact on individuals' level of welfare (longevity, good health, good nutrition, education, being well integrated into society, etc.). Thus, a new poverty measure is called for—one that accounts for other welfare indicators, particularly:

1. An indicator that accounts for a short lifespan. Denoted HPI_1, this reflects the percentage of individuals whose life expectancy is less than 40 years.

2. A measure which is related to the problem of access to education and communications. The proportion of the adult population that is illiterate, denoted HPI_2, could be considered as an appropriate indicator.

3. A composite index capturing facets of the level of material welfare, HPI_3. This is computed as the arithmetic mean of three indicators, to wit: the percentage of the population having access to healthcare (denoted $HPI_{3.1}$) and safe water ($HPI_{3.2}$), and the percentage of children under age five suffering from malnutrition ($HPI_{3.3}$).

The proposed composite poverty index was elaborated by Arnand and Sen (1997). It is written as follows:

$$HPI = (w_1 HPI_1^{\theta} + w_2 HPI_2^{\theta} + w_3 HPI_3^{\theta})^{\frac{1}{\theta}}, \tag{1}$$

with $w_1 + w_2 + w_3 = 1$ and $\theta \geq 1$.

When $\theta = 1$, the three elements of HPI are perfect substitutes. However, when θ tends to infinity, this index approaches the maximum value of its three components, i.e. $\max(HPI_1, HPI_2, HPI_3)$. In this event, the HPI will only fall if its highest-valued component decreases. These two extreme cases are difficult to advocate, so an intermediate value is sought for ordinal comparisons of poverty.[1]

The HPI omits the monetary dimension of poverty, which is at least as important as the aspects this index captures. Furthermore, this index does

[1]This methodology was notably applied by Collicelli and Valerii (2001). The results of their analysis reveal that some countries do indeed have a low poverty incidence combined with a high value of HPI. Moreover, Durbin (1999) suggests calculating a sex-based HPI to enable comparing female and male poverty.

not account for the correlation that may exist between its three components. Thus, an illiterate individual whose life expectancy is less than 40 years will be doubly counted. Finally, ordinal comparisons of poverty will be very sensitive to the (arbitrary) values assigned to w_i and θ.[2] An alternative approach that allows for a better characterization of the weights assigned to each chosen attribute would certainly be more appropriate.

The problem of choosing an appropriate weighting system for different welfare indicators was broached by Ram (1982).[3] According to him, the data must be allowed to determine the optimal weight associated with each attribute, and the *Principal Components Analysis* (PCA) method is thus appealing. Collicelli and Valerii (2000–2001) applied this procedure and constructed several multidimensional poverty indices, obtained by combining various individual welfare indicators (monetary and non-monetary).[4]

To achieve this, they derived from the available attributes new ones, called factors.[5] These factors represent all the original variables in the form of synthetic indices, and are obtained as a linear combination of the original variables. The system of weights associated with the original attributes is derived so as to reproduce the full range of variability of the latter. The "factor" variables are uncorrelated, each representing a particular aspect of

[2]We can also fault the components of the HPI index for not satisfying Sen's monotonicity axiom (1976).

[3]In this article, Ram (1982) critiques the approach proposed by Moris (1979), who suggests an index of the quality of human life that attributes the same weight to illiteracy rates, infant mortality, and life expectancy at birth. Using the PCA, Ram prefers to assign a weight of 0.4 to the first attribute, 0.32 to the second, and 0.28 to the third.

[4]See also Maasoumi and Nickelsburg (1988).

[5]This is possible using factorial analysis, which is compatible with the PCA method.

the phenomenon of poverty. Ordinal comparisons of poverty levels are thus performed using each of these factors. This allows two goals to be attained simultaneously: on the one hand, gathering the available information into synthetic indices and, on the other hand, identifying the many dimensions contributing to the poverty level in each country so as to better capture regional disparities.

Several attributes were selected for an empirical application. Some reflect monetary aspects (GDP per capita, the GINI coefficient), others capture access to education (the illiteracy rate, public expenditure on education as a percentage of GDP), and health (the infant mortality rate, life expectancy at birth). The results show that the factor that captures the greatest variability assembles some Latin American and North African countries together in an intermediate position between the countries of the OECD and those of Sub-Saharan Africa.

In using several aggregate indices, Collicelli and Valerii's (2000–2001) method does not solve the problem of double counting. This can only be achieved using individual data, which we look at in the following section.

2.2 Poverty Measures Based on Individual Data

A simple way of dealing with the multidimensional aspect of poverty consists of assuming that individuals' various attributes can be aggregated into a single indicator of welfare. Poverty can then be defined with respect to this indicator. In other words, individuals will be considered poor if their global welfare index falls below a certain poverty line, the specification of which accounts for the multidimensional aspects of poverty.

This procedure is found in Smeeding et al. (1993), in particular. They start from the simple premise that individuals' welfare depends not only on monetary income, but also on their access to certain social services, such as education and healthcare. Furthermore, when they own their homes, individuals benefit from the services their residences provide. Consequently, imputing the same level of welfare to two individuals with the same income, one of whom owns his own home while the other rents, has the net effect of underestimating the welfare level of the homeowner.

To incorporate this element, Smeeding et al. (1993) impute a value to the service homeownership confers, using either the market value of a rental, when available, or the yield on the capital market of an equivalent investment when the market value of an equivalent residence is unknown.

As to education and healthcare services, the imputed global values are assumed equal to the amount the government spends on them. The distribution across households of education services is obtained by estimating the per capita cost of primary, secondary, and university education. Expenditures on education are thus allocated according to the number of individuals in each household having completed a certain level of education.

Finally, as to the distribution of healthcare spending, Smeeding et al. (1993) treat healthcare spending as an insurance benefit received by all individuals, regardless of their actual use of these services. These benefits vary by age and sex. The value of the benefits imputed to households is thus estimated as a function of healthcare expenditures by age and sex for each group in the population.

This method was used to compare the incidence of poverty between certain OECD countries. A poverty line was set at 50 per cent of the median income (before imputing non-market services in each of the selected countries). This study yielded two important results. First, the incidence of poverty diminished in all countries with the move from the distribution of current income to the distribution of income incorporating services rendered by housing (in the case of homeowners) and some non-market services received.[6] Second, the ordinal ranking of some countries changed depending on which distribution was used. For example, Great Britain placed in the middle of the ranking for the current income distribution, but became the country with the lowest incidence of poverty when some non-market services were incorporated.

Though it constitutes an interesting attempt to account for non-market aspects of welfare, the approach applied by these authors presents certain limitations, particularly:

The value attributed by mostly poor households to non-market services may be below the cost of producing these services, in which case this method overestimates the welfare gain they provide.

This method does not preclude the possibility of compensation between different dimensions of welfare. For example, assume that there are two households equivalent in all but one dimension: one has a member who has not yet completed her university studies, while the other has a member of the same age who has just graduated (and who is seeking work). Assume further that the per capita income of both households is very near the poverty line

[6]This result is partially attributable to the fact that the same poverty line is used to compare both distributions.

before the value of non-market services is factored in. Thus, before imputing the value of non-market services, both households are considered poor, but imputing the cost of university education means that the first household is no longer poor, while the second remains poor. It is, however, far from certain that the welfare level of the first is higher. A poverty line that is specific to the needs of the household would have avoided this problem.

The approach implemented by Pradhan and Ravallion (2000) solves this problem of overestimating benefits resulting from incorporating government services. It constitutes a multidimensional extension of the subjective evaluation of welfare in general and the poverty line in particular.[7] This evaluation is based on the following question addressed to households: "What income level do you personally consider to be absolutely minimal? That is to say that with less you could not make ends meet?" The same question can be asked for each attribute in a multidimensional analysis.

Derivation of the subjective poverty line for each attribute can be facilitated by using the following model:

$$\ln z_{ij} = \delta_j + \mu_j \ln x_{ij}, \tag{2}$$

where z_{ij} is the subjective poverty line for attribute j revealed by individual i, and x_{ij} is the level of expenditure on that attribute. When the elasticity of the subjective poverty line with respect to expenditure on each attribute is less than one, the minimum required for j to be socially acceptable is given

[7] For the subjective evaluation of welfare see, for example Kapteyn (1994) and Kapteyn et al. (1988).

by[8]

$$z_j^* = exp\left(\frac{\delta}{1-\mu_j}\right). \tag{3}$$

The global subjective poverty line is defined as the least expenditure required for an individual to be able to acquire the minimum of each attribute. An individual is thus considered poor when his income falls below the subjective poverty line,

$$z^* = \sum_{j=1}^{j=k} z_j^*. \tag{4}$$

Pradhan and Ravallion (2000) applied this approach to microdata from Nepal and Jamaica. Their initial goal was to consider food consumption, clothing, housing, transportation, children's schooling and healthcare, education, and healthcare. However, in the empirical implementation, the last three attributes were omitted. The principal result of this analysis is that subjective measures of poverty (such as the incidence of poverty and the normalized poverty deficit) are greater than measures based on official estimates of the poverty line.

The Pradhan and Ravallion (2000) approach certainly contributes a great deal to integrating multidimensionality, especially should it prove possible to resolve difficulties associated with accounting for attributes omitted from their study. Nonetheless, it remains very restrictive and, ultimately, amounts to reducing the multidimensional aspect of poverty to a single dimension,

[8] It should be noted that Pradhan and Ravallion (2000) did not use this model to estimate the subjective poverty line. In fact, they did not have a subjective value for z_{ij} for each individual. Rather, they had a score from one (1) to four (4) indicating whether a household was not at all satisfied with its situation ($score_{i,j} = 1$) or very satisfied ($score_{i,j} = 4$). To determine the subjective poverty line they used an ordered probit.

with a more apt generalization of the concepts of income and the poverty line.

Klasen (2000) developed an alternative approach in order to avoid the difficulties encountered when including certain attributes in the analysis of poverty. He assigned a score from one (1) to five (5) to each attribute.[9] When the score of an attribute j for individual i is equal to one (1), i.e. $x_{i,j} = 1$, the individual is in a position of extreme deprivation with respect to this attribute. Conversely, if $x_{i,j} = 5$, the individual is very comfortablye with regard to this attribute.

In order to aggregate the scores for each individual, Klasen (2000) proceeded as follows:

$$\overline{x_i} = \sum_{j=1}^{j=k} w_j x_{i,j}. \tag{5}$$

To determine the weight, w_j, assigned to each attribute, two methods are used. The first consists of computing the mean of the scores by assigning the same weight to all attributes $\left(w_j = \frac{1}{k}\right)$. The second relies on the Principal Component Analysis (PCA) method to derive the different weights.[10] Next, two global poverty lines are computed, respectively corresponding to the

[9] The notion of assigning a score to different attributes in order to avoid basing the analysis only on a monetary indicator is not completely new. For example, Townsend (1979) let a score equal zero (0) when a household was satisfied with its endowment and one (1) otherwise. From a selection of twelve attributes, he considered a total score equal to six to represent extreme destitution of the individual. Nolan and Whelan (1996) used factorial analysis to group highly correlated attributes into a single "factor," each of which contained information about a particular dimension of poverty.

[10] Application of this procedure to South African microdata reveals that the results yielded by these two methods are very similar.

mean of the individual situated at the 20th (for extreme poverty) and at the 40th percentile of the distribution of the $\overline{x_i}$, ranked in ascending order. Also, two monetary poverty lines are calculated using the same way.

The poverty incidence and deficit are computed for different subgroups of the population, differentiated by household size, place of residence, level of education, etc. Comparing the extent of one-dimensional (or monetary) and multidimensional poverty within the different subgroups reveals, for example, that households living in urban areas are less affected by multidimensional poverty, but more by monetary poverty.

Like the Pradhan and Ravallion (2000) method, that of Klasen (2000) does not preclude compensation between attributes. Thus, if an individual's score on the first attribute is five (5) while that on the second is one (1), she will not be considered poor if the poverty line is below three (3), despite being in a position of extreme deprivation with respect to the first attribute. Furthermore, the method by which scores are attributed is very arbitrary.

Starting from Sen's (1992) capabilities approach, which seeks to identify households unable to develop the capabilities required for a decent life. Haverman and Bershadker (2001) propose a new conception of poverty based on households' skill in capitalizing on their own resources (physical and intellectual) to escape from poverty. The poverty measure yielded should identify those households that have the greatest difficulty, i.e. those at the bottom of the distribution of "*capabilities-to-generate-minimum-necessary-income.*" They call this measure "*self-reliant poverty.*" Individuals who are chronically poor are unable to be economically independent. They cannot generate an income exceeding that deemed the minimum required according to the standards of the society under consideration.

The motivation for this new poverty measure is obvious, according to Haverman and Bershadker (2001). Indeed, the state of being unable to reach the minimum income required to cover the basic needs indicates a situation that is much more serious than that of individuals who are short of money owing to a downturn in the business cycle or because they are looking for a better job, than that of those who are transiently poorly housed, or even that of those whose consumption is temporarily below the minimum required. Moreover, identifying households that are poor, but are nonetheless able to escape from poverty by their own efforts, is absolutely vital. Transfers targeted at these households must be time-limited, so that they do not become dependent on social assistance.

To measure "*self-reliant poverty*," Haverman and Bershadker (2001) begin by measuring the capability of each adult living in the household to earn an annual income. This estimated income corresponds to the amount an adult should earn if she worked full-time for one year earning a wage commensurate with her physical and intellectual capabilities.[11] This, then, yields the household's capacity to generate income. If this income falls below the official poverty line, the household is deemed unable to be economically independent, even if all adult members work full-time.

Application of this methodology to U.S. data reveals that "*self-reliant poverty*" is growing faster than the incidence of poverty. It also reveals that

[11]To estimate this income, the authors regressed the log of observed income on the variables affecting the wage rate (the level of education, age, health), incentives to work (non-labour income, the number of dependent children), and labour-market conditions (the unemployment rate).

single-parent families and families with little human capital are most affected by this poverty.

Clearly, this approach only partially reflects Sen's capabilities approach (1992). Indeed, this approach does not account for the deprivation suffered by families with limited access to some public services. Moreover, Haverman and Bershadker's (2001) empirical results are somewhat surprising. In fact, they show that approximately half of the households that are "*self-reliant poor*" are not poor in terms of their observed incomes. Are these families temporarily not poor? Or have estimation errors caused them to be classified as *self-reliant poor*?

3 An Axiomatic Approach to Elaborating Multidimensional Poverty Measures

Measuring poverty always raises ethical questions. For example, should we consider a person who is well endowed with some attributes poor if she is unable to attain the minimum requirements for one basic need? The answer is not obvious. It would appear reasonable to consider an individual poor if her life expectancy falls below a certain threshold, even if her income is quite high. The same logic can be applied to an individual whose life expectancy is long, even if his income is below the minimum required. Some of the approaches discussed above implicitly reflect the opposite point of view. In fact, when it is possible to assign a virtual price p_j to each attribute j, an individual will not be considered poor if $\sum p_j x_{i,j} \geq \sum p_j z_j$, suggesting that attributes are perfectly substitutable!

It is clear that the diversity of opinions springs from the fact that poverty is not an objective concept. Rather, it is a complex notion, the normative analysis of which may be facilitated by adopting an axiomatic approach. This emphasizes the desirable properties (axioms) that a poverty index must respect. These axioms, though they allow us to characterize measures of poverty, may make any agreement on the analysis results even more remote (Section 3.1). Some recent studies have sought to establish the necessary conditions for ordinal comparisons of welfare distributions to be robust, that is valid for a large choice of poverty lines and poverty measures (Section 3.2).

3.1 Presentation of the Principal Axioms and the Measures They Yield

The most general form of a class of multidimensional poverty measures can be given by the following equation:

$$P(X, z) = F\left[\pi\left(x_i, z\right)\right], \tag{6}$$

where $\pi(\cdot)$ is an individual poverty function that indicates how the many aspects of poverty must be aggregated at the level of each person. The function $F(\cdot)$ reflects the way in which individual poverty measures are aggregated to yield a global measure of poverty. For example, if the function $F(\cdot)$ is additive, we have

$$P(X, z) = \frac{1}{n}\sum_{i=1}^{n}\pi(x_i, z), \tag{7}$$

and, if $\pi(\cdot)$ is an index function such that

$$\pi\left(x_i, z\right) = \begin{cases} 0, & \text{if } x_{i,j} \geq z_j, \quad \forall j = 1, 2, \ldots, k, \\ 1, & \text{otherwise,} \end{cases}$$

we have a multidimensional extension of the incidence of poverty.[12]

Generally, the properties of $F(\cdot)$ and $\pi(\cdot)$ will depend on the axioms that the poverty measures are stipulated not to violate. Some axioms having been developed in the literature on multidimensional poverty measures are new, but others are simply generalizations of those inherent in the construction of one-dimensional poverty measures.

Given the difficulty of obtaining precise data on fundamental needs, we may reasonably require that a poverty measure be continuous with respect to them.[13] This circumvents the problem of small errors of measurement causing draconian changes in poverty readings. The following axiom fulfills this requirement:

Axiom 1 *Continuity: The poverty measure must not be sensitive to a marginal variation in the quantity of an attribute.*

Individuals' identity, or any other indicator that is irrelevant to the analysis of poverty, must not have any impact on the results of the analysis. This principle is summed up in the following proposition:

Axiom 2 *Symmetry (or Anonymity): All characteristics other than the attributes used to define poverty do not impact on poverty.*

Generally, ordinal poverty comparisons occur between populations of different sizes, whence the necessity of this axiom:[14]

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[12] Unlike the HPI index, this measure does not double count poor individuals for each attribute.

[13] See, for example, Donaldson and Weymark (1986).

[14] This axiom was introduced into poverty analysis by Chakravarty (1983) and Thon

Axiom 3 *The Principle of Population: If a matrix of attributes is replicated several times, global poverty remains unchanged.*

Similarly, different countries that are subject to an ordinal comparison of poverty may use different units of measure. Consequently, any poverty index should be independent of the units of measure. The following axiom expresses this requirement:[15]

Axiom 4 *Scale Invariance: The poverty measure is homogeneous of degree zero (0) with respect to X and z.*

This axiom makes it clear that the individual poverty function will have the following form:

$$\pi\left(x_i, z\right) = \pi\left(\frac{x_{i,1}}{z_1}, \ldots, \frac{x_{i,j}}{z_j}, \ldots, \frac{x_{i,k}}{z_k}\right). \tag{8}$$

Axiom 5 *Focus: The poverty measure does not change if an attribute j increases for an individual i characterized by $x_{i,j} \geq z_j$.*

(1983). One of its consequences is that the poverty measure falls with increases in the size of the non-poor population. Henceforth, the Focus 2 axiom requires that the poverty measure be independent of the distribution of attributes among the non-poor, while the population principle requires a decreasing relationship between the size of this population and the poverty measure.

[15] Blackorby and Donaldson (1980) distinguish this axiom from another, Transformation Invariance. This suggests that

$$P\left(X + T, z + t\right) = P\left(X, z\right).$$

We have not retained this latter axiom, because it has only been used by Tsui (2002) in a multidimensional analysis.

Using this axiom, we should find:

$$\frac{\partial \pi}{\partial x_{i,j}} = 0 \text{ if } x_{i,j} \geq z_j. \tag{9}$$

Thus, the isopoverty curves for a poor individual run parallel to the axis of the j-th attribute when $x_{i,j} \geq z_j$.[16] The following axiom reveals that the multidimensional incidence of poverty (as given by the HPI index, for example) is not completely satisfying in some respects:

Axiom 6 *Monotonicity: The poverty measure declines, or does not rise, following an improvement affecting any of a poor individual's attributes.*[17]

The consequence of this axiom is that isopoverty curves are not increasing, i.e.

$$\frac{\partial \pi (x_i, z)}{\partial x_{i,j}} \leq 0 \text{ if } x_{i,j} < z_j. \tag{10}$$

As is the case for one-dimensional measures, it is desirable that multidimensional poverty measures be sensitive to the welfare levels of different segments of the population with homogeneous characteristics, such as age, sex, place of residence, etc. The following axiom spells out this property for a situation in which the total population can be decomposed into two subgroups (called a and b):

[16] An iso-poverty curve indicates the various vectors x_i that yield the same level of individual poverty, i.e. $\pi (x_i, z) = \bar{\pi}$.

[17] For example, the multidimensional poverty incidence and the HPI index may violate this axiom. Indeed, if malnutrition becomes worse among children already affected by that problem, the value of the multidimensional poverty incidence and the HPI index remain unchanged.

Axiom 7 *Subgroup Consistency: Let* $X\begin{bmatrix} X^a \\ X^b \end{bmatrix}$ *and* $Y\begin{bmatrix} Y^a \\ Y^b \end{bmatrix}$ *with* X^a *and* Y^a *(*X^b *and* Y^b*) being* $n^a \times k$ $\left(n^b \times k\right)$ *matrices. If* $P\left(X^a, z\right) > P(Y^a, z)$ *while* $P\left(X^b, z\right) = P\left(Y^b, z\right)$*, then*

$$P\left(X, z\right) > P\left(Y, z\right).$$

A multidimensional measure of poverty obeys the preceding axiom if it can be formulated as follows:

$$P\left(X, z\right) = F\left[\sum_{i=1}^{n} \frac{1}{n} \pi\left(x_i, z\right)\right]. \tag{11}$$

When $F\left(\cdot\right)$ is additive, the poverty measure $P\left(X, z\right)$ also respects the decomposability axiom:

Axiom 8 *Subgroup Decomposability: Global poverty is a weighted mean of poverty levels within each subgroup:*

$$P\left(X, z\right) = \sum_{s=1}^{S} \frac{n_s}{n} P\left(X^s, z\right).$$

Poverty measures that satisfy *Decomposability* enable the evaluation of each population segment's contribution to global poverty. This makes possible the conception of poverty-fighting programs that are more focussed on the most vulnerable.[18]

The literature dealing with multidimensional poverty distinguishes between measures based on the union of the various aspects of deprivation from those based on their intersection.[19] Chakravarty et al. (1998) opt for

[18] More detail on the usefulness of this type of axiom can be found in Bourguignon and Chakravarty (1998), Chakravarty et al. (1998), and Tsui (2002).

[19] More information about this is presented in Atkinson (2002) and Duclos et al. (2002).

measures based on the union. In addition to decomposing the population by subgroup, they also propose a decomposition by attribute:

Axiom 9 *Factor Decomposability: Global poverty is a weighted mean of poverty levels by attribute.*[20]

According to Chakravarty et al. (1998) and Bourguignon and Chakravarty (1998), this double decomposition makes easy the design of inexpensive and efficient programs to combat poverty. It is thus particularly useful when financial constraints preclude the elimination of poverty in an entire population segment or by a specific attribute. If the double decomposition is retained, then multidimensional poverty measures take the following form:

$$P(X, z) = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{k} \pi_j (x_{i,j}, z_j), \tag{12}$$

and the condition

$$\frac{\partial^2 \pi (x_i, z)}{\partial x_{ij} \partial x_{i,j}} = 0 \tag{13}$$

will automatically be met.

In the event that $\pi (x_i, z_j)$ assumes one of the following two forms:

$$\pi (x_i, z) = \sum_{j=1}^{k} a_j \left(\frac{z_j - x_{i,j}}{z_j} \right)^{\alpha}, \tag{a}$$

or

$$\pi (x_i, z) = \sum_{j=1}^{k} a_j \ln \left[\frac{z_j}{\min (x_{i,j}, z_j)} \right], \tag{b}$$

we obtain a multidimensional extension of the FGT poverty measure (in the first case) and that of Watts (1968) (in the second), satisfying all the

[20] Bourguignon and Chakravarty (1998) show that, under certain conditions, a decomposition by factors necessarily arises.

preceding axioms. Conversely, the following multiplicative extension to the FGT class:

$$\pi(x_i, z) = \prod_{j=1}^{j} \left(\frac{z_j - x_{i,j}}{z_j} \right)^{\alpha_j}, \tag{c}$$

where α_j is a parameter reflecting poverty aversion with respect to attribute j, does not respect decomposability by factor. Moreover, in this case, poverty is measured across the intersection of various dimensions of human deprivation. In fact, an individual having the minimum required for a single attribute, but less than the minimum for all others, will not be considered part of the population of the poor.

Factor Decomposibility necessarily leads to poverty measures based on the union of different dimensions of poverty—but the opposite is not always the case. For example the Tsui (2002) index, though not compatible with Factor Decomposability, is based on the union of the various dimensions of poverty:[21]

$$\pi(x_i, z) = \prod_{j=1}^{j} \left[\frac{z_j}{\min(x_{i,j}, z_j)} \right]^{\beta_j} - 1. \tag{d}$$

Sen (1976) suggests that poverty measures should be sensitive to inequalities within the poor population. In other words, a Dalton transfer from a relatively less poor individual to a poorer one should reduce the poverty index.[22] This principle was applied by Kolm (1977) to study the problem of inequality in a multidimensional context. For a multidimensional poverty measure, Tsui (2002) introduced the following axiom:

[21] This is a multidimensional extension of Chakravarty's (1983) measure. Aside from decomposability by factor, this measure obeys all the axioms developed so far.

[22] Dalton (1920) observed that a transfer from a non-poor individual to a poor one improves social welfare as long as there is no reclassification of the two individuals.

Axiom 10 *Transfer: Poverty is not increased with matrix Y if it is obtained from matrix X by simply redistributing the attributes of the poor using a bistochastic transformation (and not permutation) matrix.*[23]

Intuitively, the distribution reflected by matrix Y is more egalitarian than that in matrix X if extreme solutions are replaced with more mid-range solutions. For example, assume two attributes such that $z_1 = 10$ and $z_2 = 12$. Let the initial distribution be characterized by $x_1\,(2, 10)$ and $x_2\,(8, 2)$. If Y is obtained from X using a bistochastic matrix B all of the elements of which are equal to 0.25, the two individuals will have $y_1\,(5, 6)$ and $y_2\,(5, 6)$, respectively. Clearly, the distribution Y is more egalitarian than X, which explains why it must contain less poverty. Thus, this property implies that the isopoverty curves must be convex, or

$$\frac{\partial^2 \pi(x_i, z)}{\partial x_{ij} \partial x_{i,j}} \geq 0, \quad \forall x_{i,j} < z_j. \tag{14}$$

We can confirm that the axiom of Transfer is satisfied by the Watts (1968) measure, the FGT measures when $\alpha > 1$, and the Tsui (2002) measures when $\beta_j > 0$.

There is an inequitable type of transfer that is not covered by the preceding developments. Assume that $k = 2$, $z_1 = 8$, and $z_2 = 6$ (where z_1 represents the minimum education requirement and z_2 the minimum income requirement). Let $x_1\,(2, 1)$, $x_2\,(3, 5)$, and $x_3\,(7, 2)$, and assume that after a transfer we have $y_1\,(2, 1)$, $y_2\,(3, 2)$, and $y_3\,(7, 5)$. The correlation between the attributes increases subsequent to this transfer, i.e. an individual having more of one attribute also has more of the other attribute. Intuitively,

[23] The values of the elements of a doubly stochastic transformation matrix are between zero (0) and one (1). Each row (column) of such a matrix sums to one (1).

poverty must increase, or at least not decrease, after this type of transfer.[24] The following axiom, proposed by Tsui (2002), imposes that a poverty measure should not decrease after this type of transfer:

Axiom 11 *Nondecreasing Poverty Under a Correlation Increasing Switch: Let Y be obtained from X following a series of transfers within the poor population. Let these transfers increase the correlation between attributes while no individual actually ceases to be poor, then*

$$P(X,z) \geq P(Y,z).$$

Bourguignon and Chakravarty (1998) point out that this axiom is valid for substitutable attributes. In this situation, substitutability must be understood in terms of closeness in the nature of the attributes. In light of this, if we let education and income be two attributes with similar natures in the preceding example, then the poverty of individual 3 does not decline by very much when income increases, because her education level is high. The decrease would have been greater had she been less educated. It is important that the expected fall not offset the increase in poverty of individual 2, whose income has decreased while his education level is low. Analytically, when attributes are substitutable, we have

$$\frac{\partial^2 \pi(x_i, z)}{\partial x_{ij} \partial x_{i,k}} \geq 0, \quad \forall x_{i,j} < z_j. \tag{15}$$

We must conclude that poverty measured by twice-decomposable indices will remain unchanged subsequent to any transfer increasing the correlation

[24] Atkinson and Bourguignon (1982) suggest that a measure of social welfare must not increase after this type of transfer.

between attributes. Henceforth, this last axiom will always be (weakly) satisfied by this type of measure. The Tsui (2002) poverty measure will necessarily increase if $\beta_j \beta_k > 0$.

However, when two attributes are considered complementary, the fall in poverty of individual 3 must be greater, at least to the point of compensating for the increase in poverty of individual 2. The following axiom, introduced by Bourguignon and Chakravarty (1998), generalizes the preceding one:

Proposition 1 ***Axiom 12*** *Poverty is nondecreasing (nonincreasing) subsequent to a rise in the correlation between two attributes when these attributes are substitutes (complements).*

Analytically, when the attributes are complements, we have

$$\frac{\partial^2 \pi(x_i, z)}{\partial x_{ij} \partial x_{i,k}} \leq 0, \quad \forall x_{i,j} < z_j. \tag{16}$$

Bourguignon and Chakravarty (1998) propose an extension to the FGT class of measures that, in addition to respecting all the axioms developed above, also allows for substitutability and complementarity among attributes:[25]

$$P_{\alpha,\gamma}(X, z) = \frac{1}{n} \sum_{i=1}^{n} \left[\left(\frac{z_1 - x_{i,1}}{z_1} \right)^{\gamma} + b^{\frac{\gamma}{\alpha}} \left(\frac{z_2 - x_{i,2}}{z_2} \right)^{\gamma} \right]^{\frac{\alpha}{\gamma}}, \tag{17}$$

where $\alpha \geq 1$, $\gamma \geq 1$, and $b > 0$. $\alpha \geq 1$ ensures that the transfer principle for a single attribute is respected for poor people. When $\alpha \geq 1$, $\gamma \geq 1$ ensures that this principle extends to individuals who are poor in two attributes simultaneously. As the value of γ increases, the isopoverty curve becomes more convex. The elasticity of substitution between the two poverty deficits

[25]To keep the presentation tractable, we use the $k = 2$ case.

is $\frac{1}{\gamma-1}$. The (positive) magnitude of b reflects the relative weight of the second attribute vis-à-vis the first. When $\alpha \geq \gamma \geq 1$, the two attributes are substitutes and the measure given by $P_{\sigma,\gamma}(X,z)$ respects the axiom that *poverty is nondecreasing after an increase in correlation between the attributes.* Conversely, when $\gamma \geq \alpha$, the two attributes are complements, and $P_{\alpha,\gamma}(X,z)$ satisfies the condition that poverty is nonincreasing subsequent to a rise in the correlation between the two attributes. When $\gamma = 1$, the isopoverty curves are linear for these two attributes in the case of poor individuals. Finally, as the value of γ becomes very large, the measure $P_{\alpha,\infty}(X,z)$ can be written as follows:

$$P_{\alpha,\infty}(X,z) = \frac{1}{n}\sum_{i=1}^{n}\left[1 - \min\left(1, \frac{x_{i,1}}{z_1}, \frac{x_{i,2}}{z_2}\right)\right]^{\alpha}. \tag{18}$$

In this case, the two attributes are complementary and the isopoverty curves assume the shape of Leontief curves.

3.2 Robustness Analysis

Because ordinal poverty measures are liable to be mitigated by an alternate choice of z or $P(X,z)$, the stochastic dominance approach seeks to establish the conditions under which comparisons remain valid for a plausible range of variation of z and for a given family of poverty measures. The principal results of stochastic dominance theory in a single dimension are:[26]

[26]For more information on using stochastic dominance theory to establish an ordinal ranking of welfare distributions based on one-dimensional poverty measures, see Atkinson (1987), Foster and Shorroks (1988), Jantti and Danziger (2000), and Jenkins and Lambert (1997).

Poverty decreases, or does not increase, for any possible choice of $z_j \in \left[0, z_j^*\right]$, when moving from a distribution A to a distribution B of attribute j, if the incidence of poverty under distribution A is never greater than that under distribution B. If this condition is observed, then the condition for first-order stochastic dominance holds. Otherwise, it is possible to establish a weaker condition, that of second-order stochastic dominance. This requires that poverty, as measured by the normalized poverty deficit, does not increase for any possible choice of $z_j \in \left[0, z_j^*\right]$, when moving from a distribution A to a distribution B.

While the literature dealing with issues of dominance in a one-dimensional environment (based on an axiomatic approach) is well developed, research into the multidimensional aspect is scarcely beginning, and remains an important avenue of exploration.

Bourguignon and Chakravarty (2002) seek to establish conditions for the robustness of a given ordinal ranking, given X and z, under the assumption that the upper poverty line for each attribute remains fixed. They also assume that the poverty measure respects the *axioms of Focus, Symmetry, Principle of Population, and Subgroup decomposability.*

For $k = 2$, the distribution of attributes $x_i\left(x_{i,1}, x_{i,2}\right)$ is replaced by the cumulative distribution function $H\left(x_1, x_2\right)$, defined on $[0, a_1] \times [0, a_2]$. The goal is to compare two distributions: H and H^*. Given the *axiom of decomposability*, poverty associated with the distribution H can be written as:

$$P(H, z) = \int_0^{a_1} \int_0^{a_2} \pi_z(x_1, x_2) dH, \tag{19}$$

where $\pi_z\left(x_1, x_2\right)$ is the level of poverty associated with an individual having attributes (x_1, x_2). The poverty differential between H and H^* is given by

$$\Delta P(z) = \int_0^{a_1} \int_0^{a_2} \pi_z(x_1, x_2) d\Delta H, \tag{20}$$

where $\Delta H = H(x_1, x_2) - H^*(x_1, x_2)$. Distribution H (weakly) dominates H^* if ΔP is negative (nonpositive) for all $\pi_z(x_1, x_2)$ belonging to a given class of measures $P(\cdot)$.

Bourguignon and Chakravarty (2002) study multidimensional families of poverty measures that are in line whith the axiom of Monotonicity. They distinguish between classes of measures with two substitutable, complementary, or independent attributes. They show that substituability among attributes is associated with the intersection of many dimensions of poverty, while complementarity is related to their union. More precisely:

- When two attributes are substitutable, i.e. $\frac{\delta^2 \pi_x(x_1,x_2)}{\delta x_1, \delta x_2} > 0$, stochastic dominance requires first-order dominance in each dimension of poverty,

$$\Delta P(x_j) = \int_0^{x_j} d\Delta H_{u_j}(u_j) \leq 0, \quad \forall x_j \leq z_j, \tag{21}$$

  and first-order dominance across the intersection of the two dimensions of poverty,

$$\Delta P(x) = \int_0^{x_1} \int_0^{z_{x2}} d\Delta H(u_1, u_2), \quad \forall x_j \leq z. \tag{22}$$

- When the two attributes are complements, i.e. $\frac{\delta^2 \pi_x(x_1,x_2)}{\delta x_1, \delta x_2} < 0$, stochastic dominance also requires the first-order robustness of each dimension of poverty (Equation 21). Among other things, first-order dominance across the union of the two dimensions of poverty is required:

$$\Delta P(x) = \sum_{j=1}^{j=2} \int_0^{x_j} \Delta H_{u_j}(u_j) du_j \tag{23}$$

$$-\int_0^{x_1}\int_0^{z_{x2}} d\Delta H(u_1,u_2) \leq 0, \quad \forall x_j \leq z_j.$$

- When the two attributes are independent: i.e. $\frac{\delta^2 \pi_x(x_1,x_2)}{\delta x_1, \delta x_2} = 0$, the selected poverty measures are twice decomposable. Stochastic dominance only requires the condition described by Equation 21.

Whenever it is desirable for poverty measures to further respect the Transfer axiom, it is far from obvious that the second-order stochastic dominance results can be applied analogously. According to Bourguignon and Chakravarty (2002), the analysis of second-order stochastic dominance requires restrictions on the signs of the second and third derivatives of the poverty function. Interpretation of these restrictions is unclear in the context of multidimensional poverty. Nonetheless, if the chosen measures are additive over attributes and population subgroup, the authors show that second-order robustness simply requires that:

$$\Delta P(x_j) = \int_0^{x_j} \Delta H_{u_j}(u_j) du_j \leq 0, \quad \forall x_j \leq z_j,$$

In other words, second-order dominance [in the sense of Atkinson (1987) and Foster and Shorroks (1988)] must be observed for each attribute for all $x_j \leq z_j$.

Duclos et al. (2002) establish conditions for robustness that do not require restrictive conditions on the intervals of variation of the different z_j-s. They define the individual welfare function as:

$$\lambda(x_1,x_2) : \Re^2 \to \Re \left| \frac{\partial \lambda(x_1,x_2)}{\partial x_1} \geq 0, \ \frac{\partial \lambda(x_1,x_2)}{\partial x_2} \geq 0. \right. \tag{24}$$

They assume that an unknown poverty frontier separates the poor from the non-poor population. This frontier is implicitly defined by $\lambda(x_1,x_2) = 0$.

The set of the poor is then defined by:

$$\Lambda(\lambda) = \left\{ (x_1, x_2) \left| \lambda(x_1, x_2) \leq 0 \right. \right\}. \tag{25}$$

Consequently, a two-dimensional poverty measure satisfying the Subgroup Decomposability axiom can be written as:

$$P(\lambda) = \int \int_{\Lambda(\lambda)} \pi(x_1, x_2, \lambda) dH(x_1, x_2), \tag{26}$$

where $\pi\left(x_1, x_2, \lambda\right)$ is the contribution of an individual characterized by the pair (x_1, x_2) to global poverty. By the focus axiom, this function is

$$\begin{aligned} \pi(x_1, x_2, \lambda) &\geq 0 \text{ if } \lambda(x_1, x_2) \leq 0, \\ &= 0 \text{ otherwise.} \end{aligned} \tag{27}$$

Depending on the analytical form chosen, the function $\pi\left(x_1, x_2, \lambda\right)$ measures poverty across the intersection, the union, or an intermediate combination of the two selected dimensions.

For purposes of robustness analysis, Duclos et al. (2002) consider the following multidimensional extension of the FGT class of measures:

$$P_{\alpha_1,\alpha_2}(X, z) = \int_0^{z_1} \int_0^{z_2} \left(\frac{z_1 - x_1}{z_1} \right)^{\alpha_1} \left(\frac{z_2 - x_2}{z_2} \right)^{\alpha_2} dH(x_1, x_2). \tag{28}$$

This index plays an important role in the ordinal robust comparisons of poverty, even though it measures poverty across the intersection of the two dimensions considered. These comparisons will be based on dominance order $r_1 = \alpha_1 + 1$ in space x_1, and $r_2 = \alpha_2 + 1$ in space x_2. $P_{0,0}\left(X, z\right)$ is the bidimensional incidence of poverty, i.e. the proportion of the population that is poor in both of those attributes simultaneously. $P_{1,0}\left(X, z\right)$ aggregates the

x_1 poverty deficit of poor individuals with respect to the second attribute. $P_{1,1}(X, z)$ aggregates the products of the poverty deficits, normalized by the size of the population.

Rather than selecting arbitrary poverty lines and measures, Duclos et al. (2002) begin by characterizing a class of poverty measures, then specify the necessary conditions for a distribution, A, to dominate another, B, for all poverty measures belonging to the defined class. They first consider the following class of poverty measures:

$$\Pi_{1,1}(\lambda^*) = \left\{ P(\lambda) \left| \begin{array}{c} \Lambda(\lambda) \subset \Lambda(\lambda^*) \\ \pi(x,\lambda) = 0 \text{ if } \lambda(x_1, x_2) = 0 \\ \pi^{x_j} \leq 0, \quad \forall x_j \\ \pi^{x_j x_k} \geq 0, \quad \forall x_j, x_k, \end{array} \right. \right\}, \tag{29}$$

where π^{x_j} (π^{x_j,x_k}) corresponds to the first (cross) derivative of the function $\pi(x, \lambda)$ with respect to x_j ($x_{j,k}$). The first row of Equation 29 defines the upper limit of the two poverty lines. The second indicates that poverty measures of $\Pi_{1,1}(\lambda^*)$ are continuous all along the frontier separating the poor from the non-poor segments of the population.[27] The third row stipulates that poverty measures in this class satisfy the Monotonicity 2 axiom. Finally, the fourth row reveals that measures in this class are compatible with the axiom underlying the substitutability of attributes.[28] Depending on the choice of functional form for $\pi(x, \lambda)$, this class may include poverty measures based on the intersection, the union, or any intermediary form of the two dimensions of poverty.

[27]This naturally precludes a two-dimensional incidence of poverty.

[28]Unlike Bourguignon and Chakravarty (2002), Duclos et al. reject the axiom underlying the substitutability of attributes.

Duclos et al. (2002) show that poverty, as measured by any bi-dimensional index of the class $\Pi_{1,1}(\lambda^*)$ is lower in A than in B, if the following condition is fulfilled:

$$\Delta P_{0,0}(x_1, x_2) < 0, \quad \forall (x_1, x_2) \in \Lambda(\lambda^*). \tag{30}$$

In other words, robustness of order $(1,1)$ requires that the percentage of the population that is poor in both attributes simultaneously be smaller under distribution A, and that this obtains for all ordered pairs $(z_1, z_2) \in [0, z_1^*] \times [0, z_2^*]$. Whenever this condition holds, any poverty index of class $\Pi_{1,1}(\lambda^*)$ will indicate that there is less poverty in A than in B, regardless of whether this index measures poverty across the intersection, the union, or any intermediary specification.

It is also possible to test for higher orders of dominance for one of the two dimensions, such as $(2,1)$ or $(1,2)$, or for both simultaneously, $(2,2)$. These tests are of particular pertinence when the $(1,1)$-order dominance yields ambiguous results, i.e. when the sign of $\Delta P_{0,0}(x_1, x_2)$ is sensitive to the choice of z_j.

Hence, because it is desirable for poverty to diminish following an equalizing (Daltonian) transfer of x_1 at a given value of x_2, and that this effect is decreasing in the value of x_2, the following class of measures becomes appealing:

$$\Pi_{2,1}(\lambda^*) = \left\{ P(\lambda) \;\middle|\; \begin{array}{c} P(\lambda) \in \Pi_{1,1}(\lambda^*), \\ \pi^{x_1,x_1} \geq 0, \quad \forall x_1, \\ \pi^{x_1 x_1 x_2} \leq 0, \quad \forall x_1, x_2. \end{array} \right\}. \tag{31}$$

A necessary and sufficient condition for poverty, as measured by any index of the class $\Pi_{2,1}(\lambda^*)$, to be unambiguously lower in A than in B, is that the poverty gap in x_1 for those individuals who are poor in x_2 be smaller under A than under B, and that, for all the range variation of $z_j \in \left[0, z_j^*\right]$. Analytically, the condition for stochastic dominance of order $(2,1)$ requires that:

$$\Delta P_{1,0}(x_1, x_2) < 0, \quad \forall (x_1, x_2) \in \Lambda(\lambda^*). \tag{32}$$

If it is deemed necessary for the transfer axiom to be respected, a class of measures $\Pi_{2,2}(\lambda^*)$ should be defined. In addition to the conditions inherent in class $\Pi_{2,1}(\lambda^*)$, this primarily imposes that $\pi^{x_2,x_2} \geq 0$. The necessary and sufficient conditions for all poverty measures of class $\Pi_{2,2}(\lambda^*)$ to show an alleviation of poverty in A as compared to B is that:

$$\Delta P_{1,1}(x_1, x_2) < 0, \quad \forall (x_1, x_2) \in \Lambda(\lambda^*). \tag{33}$$

In other words, the $(2,2)$-order stochastic dominance condition must be met. In general, when a class of poverty measures $\Pi_{r_1,r_2}(\lambda^*)$ is characterized, a necessary and sufficient condition for observing the dominance condition (r_1, r_2) is that poverty, as measured by $P_{\alpha_1,\alpha_2}(x_1, x_2)$, falls for any choice of (z_1, z_2) within all the range variation of each poverty line.

4 Conclusion

There is considerable agreement that poverty is a multidimensional problem, involving a number of monetary and non-monetary handicaps. The fact that it is impossible in practice to obtain empirical observations on all these handicaps has often led researchers to reduce poverty to a one-dimensional aspect. However, since the beginning of the 1990s, data on attributes other than income have become increasingly available. The multidimensional approach is thus more than ever required to better understand the performance of a given country in the battle against poverty in all its aspects.

Once the dearth of data availability has been overcome, researchers are confronted with a new challenge: How should information reflecting the various aspects of poverty be aggregated to yield a global measure of poverty? Should this measure focus on the situation of those who are poor according to all attributes simultaneously, or should it also account for the deprivation of those who do not reach the required minimum for any one attribute?

In order to synthesize the contribution of the various approaches to measuring poverty in its various dimensions, we have distinguished between whether or not poverty measures are based on an axiomatic approach. Our goal has been to better understand the theoretical underpinnings of each approach, as well as its limitations.

It is not our intention in this paper to pass judgement on the value of the various approaches presented. Since they are not based on the same theoretical framework, these various methodologies may clearly also yield different results in terms of the ordinal ranking of poverty. Moreover, each approach may be the most appropriate in a given context. Understanding the theoretical foundations and the limitations of each one should highlight the choice of which approach to adopt depending on the circumstances and the constraints of the study to be conducted.

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