SAM Multiplier Analysis

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Social accounting matrix and multiplier analysis

Principles

The Social Accounting Matrix (SAM) is a comprehensive data system but it is not a model as such. To come to this point we must specify which variables are exogenous and endogenous and link them through a set of mathematical relations. This is exactly what is done in a proper CGE model.

The easiest manner to transform a SAM in some kind of an economic model is to assume that all the relations are of linear type and that prices are fixed (at least in the short run). In that case the SAM can be used directly to simulate the effects of shocks on some exogenous variables or accounts. This type of exercise is known as SAM multiplier analysis and can be seen as an extension of Input-Output models.

Depending of which account are set exogenous different implicit closure hypothesis are possible. Usually we will consider three accounts as potentially exogenous : the government, the rest of the world and the capital accounts. Endogenous capital account reflects some kind of internal flexibility and endogenous Rest of the World account assumes that trade is relatively free.

As Roland-Holst and Sancho (1995) report, SAMs have been used to study (i) growth strategies in developing economies (Pyatt and Round –1985, Robinson -1988), (ii) income distribution and redistribution (Pyatt and Roe –1977, Adelman and Robinson –1978, Roland-Holst and Sancho –1992), (iii) fiscal policies (Whalley and Hillaire –1987) and decomposition of activity multipliers that shed light on the circular flow of income (Stone –1981, Pyatt and Round –1979, Defourny and Thorbecke –1984, Robinson and Roland-Holst –1988).

In order to describe the main principles let us first consider a simplified Schematic Social Accounting Matrix as shown in Defourny & Torbecke (1984) :

	Endogenous account				
Receipts / Expenses	1. Factors	2. Institutions	3. Productive activities	4. Exogenous accounts	**Total**
1. Factors			T_{13}	X_1	Y_1
2. Institutions	T_{21}	T_{22}		X_2	Y_2
3. Productive activities		T_{32}	T_{33}	X_3	Y_3
4. Exogenous accounts	L_1	L_2	L_3	LX	Y_4
Total	Y_1	Y_2	Y_3	Y_4	

A_n is define as the matrix of average expenditure propensities. It can be obtained by dividing a particular element in any of the endogenous accounts by the total for the column account in which the element occurs. X_n represents the exogenous accounts and Y_n the total of each endogenous account. With those notations it comes :

$$y_n = A_n y_n + x_n = (I - A_n)^{-1} x_n = M_a x_n$$

The multiplier matrix M_a can be decomposed, as Pyatt and Round (1978) suggested, into three economically meaningful additive (or multiplicative) components: (i) a transfers matrix that picks up the net multiplier effects induced on a given set of accounts by exogenous transfers accruing to the given set; (ii) an open-loop matrix that captures the cross effects between different groups; and (iii) a closed-loop matrix detailing the multiplier effects of an exogenous inflow on an endogenous account and return to the original recipient.

First of all, let us note Â as the diagonal bloc matrix extract from the matrix A_n:

$$\hat{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix}$$

then it comes :

$$\begin{aligned} y_n &= A_n y_n + x_n = (A_n - \hat{A}_n) y_n + \hat{A}_n y_n + x_n \\ &= (I - \hat{A}_n)^{-1} (A_n - \hat{A}_n) y_n + (I - \hat{A}_n)^{-1} x_n \\ &= A^* y_n + (I - \hat{A}_n)^{-1} x_n \end{aligned}$$

Multiplying both sides by A^* and substituting for $A^* y_n$ on the left hand side now gives:

$$\begin{aligned} y_n &= A^{*2} y_n + (I + A^*)(I - \hat{A}_n)^{-1} x_n \\ &= (I - A^{*2})^{-1} (I + A^*)(I - \hat{A}_n)^{-1} x_n \end{aligned}$$

Similarly, multiplying both sides of the initial equation by A^{*2} yields :

$$\begin{aligned} y_n &= A^{*3} y_n + (I + A^{*2})(I - \hat{A}_n)^{-1} x_n \\ &= (I - A^{*3})^{-1} (I + A^* + A^{*2})(I - \hat{A}_n)^{-1} x_n \end{aligned}$$

More generally :

$$y_n = (I - A^{*k})^{-1} \left(\sum_{j=0}^{k-1} A^{*j} \right) (I - \hat{A}_n)^{-1} x_n$$

At this stage, it is worth noting that the three steps decomposition that will finally be retained reflects the sequence of substitution that corresponds to one complete cycle in the circular flow of income within the economy and thus that it is not an arbitrary choice.

If we note $M_{a1} = (I\text{-}\hat{A})^{-1}$, $M_{a2} = I+A^{*}+A^{*2}$ and $M_{a3} = (I\text{-}A^{*3})^{-1}$ it comes : $Ma=M_{a1}M_{a2}M_{a3}$

But we can also re-write Ma as :

$$Ma = \underbrace{I}_{\text{Initial injection}} + \underbrace{(M_{a1} - I)}_{\substack{\text{net contribution of}\\ \text{transfer multiplier}}} + \underbrace{(M_{a2} - I)M_{a1}}_{\substack{\text{net contribution of open loop}\\ \text{or cross-multiplier effects}}} + \underbrace{(Ma3 - I)M_{a2}M_{a1}}_{\substack{\text{net contribution of circular}\\ \text{or closed-loop multiplier effect}}}$$

Main underlying hypothesis

Several important hypothesis are underlying the SAM multiplier analysis. They stress the limits of this type of exercises. Especially we must recall that :

- Prices are supposed to remain constant in all time. This means that we implicitly assume that there exist an excess capacity of production.
- Production technology and resource endowments are given. As a result the analysis is necessarily a short term one and no dynamic of any kind can be taken into account.
- Expenditures propensities of endogenous accounts remain constant. In the basic analysis income elasticities are unitary : the prevailing average expenditure propensities in A_n are assumed to apply to any incremental injection (M_a can be seen as the matrix of average expenditure propensities). A more realistic alternative is to specify a matrix of marginal expenditure propensities corresponding to the observed income and expenditure elasticities of the different agents (M_c). Average and marginal propensities will not generally be equal for household demand but would correspond for production (as far as there is no scale effect) and for factor payments if the value added price is set as a constant mark-up over labor costs per unit of output. Expressed in another way we would generally specify :

$$\forall(i,j)\neq 32\ C_{ij}=A_{ij} \text{ and } C_{32}=\varepsilon A_{32}.$$

- A least one of the account must be exogenous. The principle of the analysis itself implies that only accounts as a whole can be supposed exogenous (and not variables or cells).

Poverty inference through multiplier analysis

Principles

We reproduce here an analysis developed by Thorbecke and Jung (1996) that permits to use the multiplier results in order to infer the effect of shocks on poverty. For this purpose let us start with the basic equations :

$$\begin{cases} dy_1 = & & A_{13}dy_3 + dx_1 \\ dy_2 = A_{21}dy_1 + A_{22}dy_2 & & + dx_2 \\ dy_3 = & + A_{32}dy_2 + A_{33}dy_3 + dx_3 \end{cases}$$

thus

$$\begin{cases} dy_1 = A_{13} dy_3 + dx_1 \\ dy_2 = (I - A_{22})^{-1} A_{21} dy_1 + (I - A_{22})^{-1} dx_2 \\ dy_3 = (I - A_{33})^{-1} A_{32} dy_2 + (I - A_{33})^{-1} dx_3 \end{cases}$$

If we note Ma_{23} the block in the matrix Ma that links households' income and productive activities in order to test the effect of sectoral growth on poverty (but the part to be considered depends clearly of the question addressed), then it come that $Ma_{23}=RD$:

$$D = (I - A_{22})^{-1} A_{21} A_{13} (I - A_{33})^{-1}$$
$$R = \left(I - (I - A_{22})^{-1} A_{21} A_{13} (I - A_{33})^{-1} A_{32}\right)$$

Now following Kakwani (1993) we have

$$dP_{ij} = \frac{\partial P_{ij}}{\partial \bar{y}} d\bar{y} + \sum_{k=1}^{l} \frac{\partial P_{ij}}{\partial \theta_{ijk}} d\theta_{ijk}$$

The distributional effect of the exogenous shock is suppose to be negligible, so that the preceding formula simplifies to the first term. This means that the distribution intra group is supposed to be constant.

The elasticity of poverty to mean income (η) can be computed as follow:

P=F(z) the poverty head count ratio (for other poverty measurement see Kakwani -1993)
z the poverty line
f(.) the probability density function of income
L(.) the Lorentz curve of the type : $L(x)=x-ax^{\alpha}(1-x)^{\beta}$

Thus, assuming that the Lorentz curve does not shifts :

$$\frac{\partial P_{ij}}{\partial \bar{y}} = -\frac{z}{\bar{y}^2 L''(P)}$$

But we know that

$$L''(P) = z \big/ \bar{y} f(z)$$

Because

$$L'(P) = z \big/ \bar{y}$$

and finally

$$\eta_{ij} = \frac{\partial P_{ij}}{\partial \bar{y}} \frac{\bar{y}}{P} = -\frac{z f(z)}{P}$$

The elasticity of poverty to mean income depends positively of the distance between the mean income and the poverty line. As a matter of fact wealthier group will have a higher elasticity.

Following the standard result of the SAM multiplier analysis it comes that:

$$d\bar{y}_i = m_{ij} dx_j$$

thus

$$\frac{dP_{ij}}{P_{ij}} = \eta_{ij} m_{ij} \left(\frac{dx_j}{\bar{y}_j} \right)$$

Using an additive decomposable aggregated poverty measure (as the FGT indicator) it comes after few calculations (see Thorbecke and Jung –1996) that :

$$\frac{dP_{ij}}{P_{ij}} = \sum_{i=1}^{m} s_i \eta_{ij} m_{ij} \left(\frac{dx_j}{\bar{y}_j} \right) = \sum_{i=1}^{m} r_{ij} s_i d_{ij} q_{ij}$$

with

s_i the poverty share of household group i out of total poverty

q_{ij} the sensitivity of the poverty measure to the change in income (poverty sensitivity effect)

d_{ij} the element (i,j) of the matrix D and $\forall (i,j)$ $r_{ij}=m_{ij}/d_{ij}$

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